Algorithmic Frontiers of Modern Massively Parallel Computation

Introduction

Ashish Goel, Sergei Vassilvitskii, Grigory Yaroslavtsev
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Modern Parallelism (Practice)

*All dates approximate*
Modern Parallelism (Theory)

- `90 BSP
- PRAM
- MUD
- MRC
- IO-MR
- MR
- MPC(1)
- MPC(2)
- Key-Complexity
- Coordinator
- Big Data
- `03 Congested Clique
- `00 Local

* Plus Streaming, External Memory, and others
Bird’s Eye View

- 0. Input is partitioned across many machines
Bird’s Eye View

- 0. Input is partitioned across many machines

Computation proceeds in synchronous rounds. In every round, every machine:
- 1. Receives data
- 2. Does local computation on the data it has
- 3. Sends data out to others
Bird’s Eye View

– 0. Input is partitioned across many machines

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Success Measures:
– Number of Rounds
– Total work, speedup
– Communication
0. Data partitioned across machines
   – Either randomly or arbitrarily
   – How many machines?
   – How much slack in the system?
Devil in the Details

0. Data partitioned across machines

1. Receive Data
   - How much data can be received?
   - Bounds on data received per link (from each machine) or in total.
   - Often called ‘memory,’ or ‘space.’
   - Denoted by $M, m, \mu, s, n/p^{1-\epsilon}$

   - Has emerged as an important parameter.
   - Lower and upper bounds with this as a parameter
Devil in the Details

0. Data partitioned across machines
   1. Receive Data
   2. Do local processing
      - Relatively uncontroversial
0. Data partitioned across machines
1. Receive Data
2. Do local processing
3. Send data to others
   - How much data to send? Limitations per link? per machine? For the whole system?
   - Which machines to send it to? Any? Limited topology?
Devil in the Details

0. Data partitioned across machines
1. Receive Data
2. Do local processing
3. Send data to others

Different parameter settings lead to different models.
- Receive $\tilde{O}(1)$, poly machines, all connected: PRAM
- Receive, send unbounded, specific network topology: LOCAL
- Receive $\tilde{O}(1)$, send $\tilde{O}(1)$, $n$ machines, specific topology: CONGEST
- Receive $s = n/p^{1-\epsilon}$ machines, all connected: MPC(1)
- Receive $s = n^{1-\epsilon}$, $n^{1-\epsilon}$ machines, all connected: MRC
- ...
Number of Rounds:
- Well established
- Few (if any?) trade-offs on number of rounds vs. computation per round

Work Efficiency
- Important!
- See “Scalability! But at What COST? [McSherry, Isard, Murray `15]

Communication
- Matrix transpose -- linear communication yet very efficient
- Care more about skew, limited by input size
Consensus Emerging:

Parameters:
- Problem size: $n$
- Per machine, per round input size: $s$

Metric:
- Number of rounds: $r(s, n)$
- Ideal: $O(1)$ - e.g. group by key
- Sometimes $\Theta(\log_s n)$: sorting, dense connectivity
- Less ideal $O(\text{poly log } n)$: sparse connectivity
Simulations

Theorem: Every round of an EREW PRAM Algorithm can be simulated with two rounds.

- Direct extensions to CREW, CRCW Algorithms

Proof Idea:

- Divide the shared memory of the PRAM among the machines, and simulate updates.
Proof Idea:

- Divide the shared memory of the PRAM among the machines. Perform computation in one round, update memory in next.

![Memory Diagram]

```plaintext
Memory: 0 1 0 0 1 0 0 0 1 0 0 0 0 1 1
```
Proof Idea:
- Have “memory” machines and “compute machines.”
- Memory machines simulate PRAM’s shared memory
- Compute machines update the state

- EREW PRAM: Every at most two outputs & inputs (one for memory, one for compute)
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EREW PRAM: Every at most two outputs & inputs (one for memory, one for compute)
Simulations

Theorem: Every round of an EREW PRAM Algorithm can be simulated with two rounds.
- Direct extensions to CREW, CRCW Algorithms

But, stronger than PRAMs.
- Subset sum. Given an array $A$, compute $B[i] = \sum_{j=0}^{i} A[j]$ for all $i$.
- Requires $O(\log n)$ rounds in PRAM
- Can be done in $O(\log_s n)$ rounds with space $s$
One Technique: Coresets!
- Reduce input size from $n$ to $s$ in parallel
- Solve the problem in a single round on one machine

Very Practical!
- $n$ : Peta/Tetabytes
- $s \approx \sqrt{n}$ : Giga/Megabytes

Talks today about coresets for:
- Clustering: k-means, k-median, k-center, correlation
- Graph Problems: connectivity, matchings
- Submodular Maximization
Some progress!

- Good bounds on what is computable in one round
- Multi-round lower bounds for restricted models (talks today)

Canonical problem:

- Given a two-regular graph, decide if it is connected or not.
- Best upper bounds $O(\log n)$ for $s = o(n)$
- Best lower bounds $\Omega(\log s n)$ by circuit complexity reductions.
  - To improve must take number of machines into consideration
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References: Models


**Key-Complexity**: Goel, Munagala. Complexity Measures for MapReduce, and Comparison to Parallel Sorting. ArXiV 2012.

**MR**: Pietracaprina, Pucci, Riondato, Silvestri, Upfal. Space Round Tradeoffs for MapReduce Computations. ICS 2012


**Big Data**: Klauck, Nanongkai, Pandurangan, Robinson. Distributed Computation of Large Scale Graph Problems. SODA 2015