Clustering in a Few Rounds

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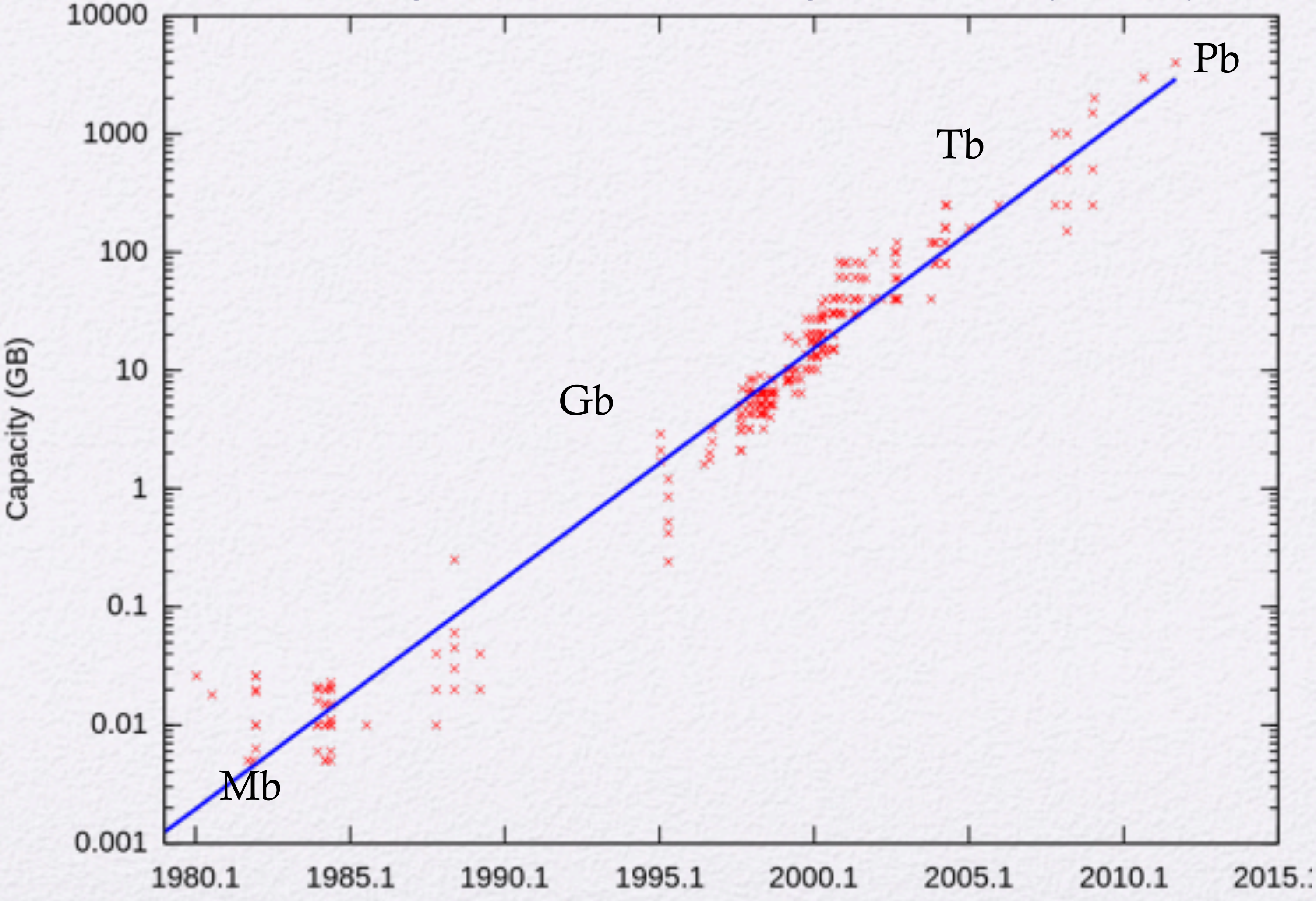
Data

"640k ought to be enough for anybody."



Graph mining challenges

- Can be implicitly defined
  - Similarities
- Nodes/edges can change
  - Social connections
- Can have special properties
  - Heavy-tailed, small-world, bipartite, ...
- Can be noisy
  - Some edges missing, some spurious

Why are graphs hard?

- Poor locality of memory access
  - Neighbors of a node can be arbitrarily located in memory
- Degree of parallelism change during execution
  - Can depend on sub-graph structures
- Nodes by themselves do not do much work
  - Edge interactions form the bulk of many graph algorithms

Graph stream

- Graph arrives as an edge stream
  - No random access to graph
  - Can be new edge or updates to existing edges
- Typically single CPU
- Very limited amount of RAM
  - Some cases, only Mb even for Tb+ data
  - May not be able to store any portion of the graph in memory
  - Graph size may be infinite/unknown in advance
- Ideally, make a single pass over the graph
  - In some cases, can take multiple rounds

Graph clustering

- How to solve large-scale clustering problems on graphs?
- Many flavors of clustering definitions
  - k-means, k-median, densest subgraphs, correlation clustering, ..
- Focus on algorithms
  - with provable guarantees
  - that run in a small number of rounds

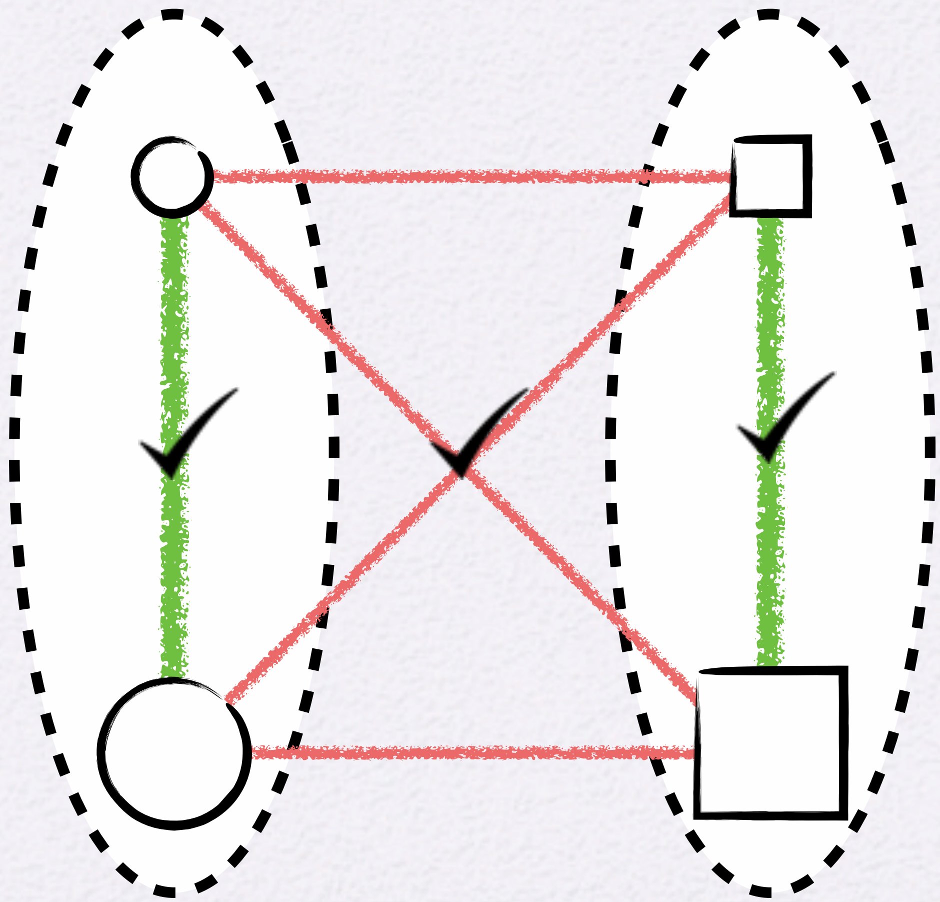
1. Correlation clustering (CC)

- Given a complete graph where each edge is +1 or -1, partition the nodes to minimize the total number of mistakes

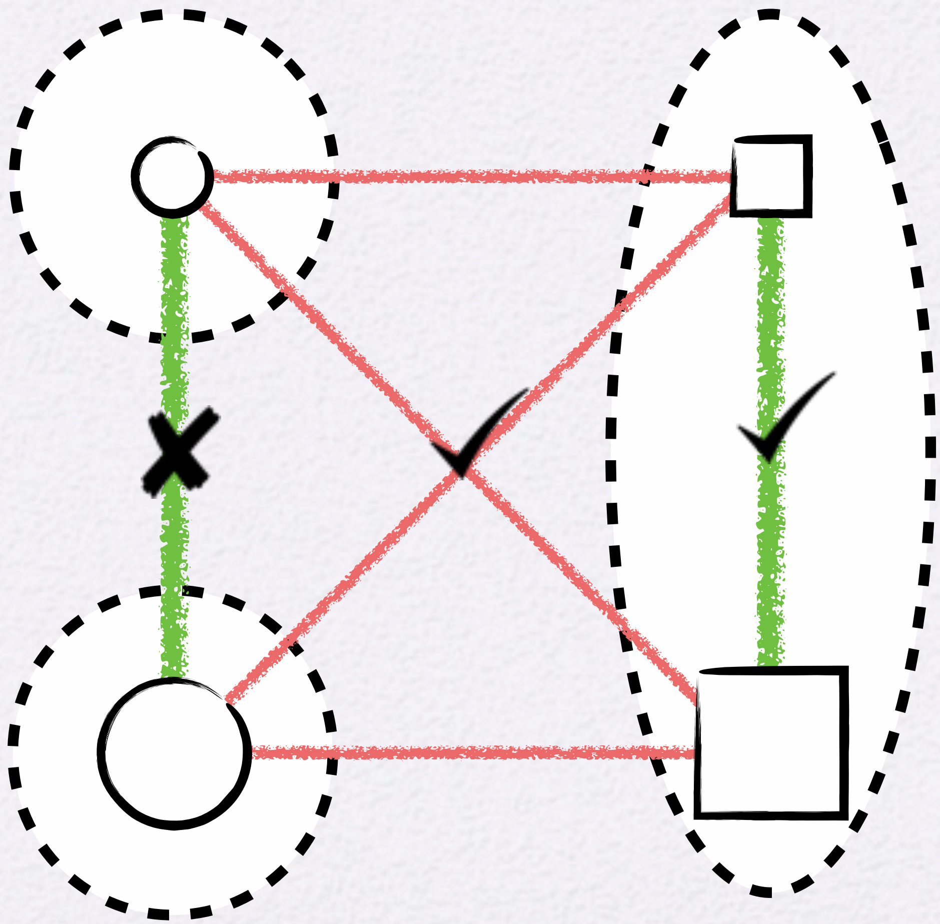
  [Bansal, A. Blum, Chawla]

- Number of clusters not specified a priori
- Often, missing edges are interpreted -1
- Machine learning / data mining applications

Eg: 0 mistakes



Eg: 1 mistake



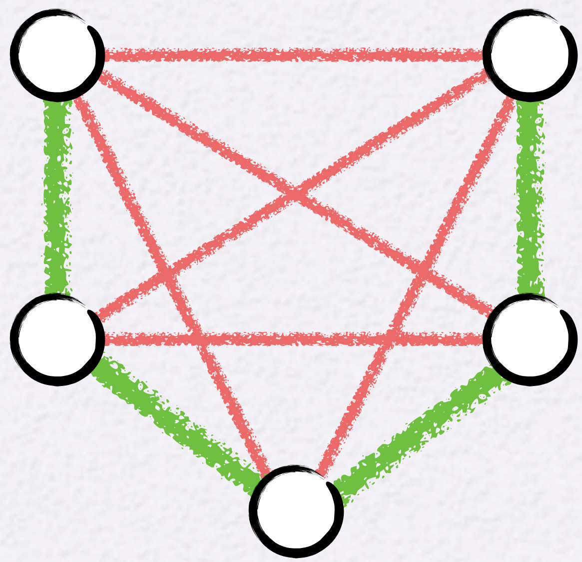
The Pivot algorithm

A simple iterative algorithm

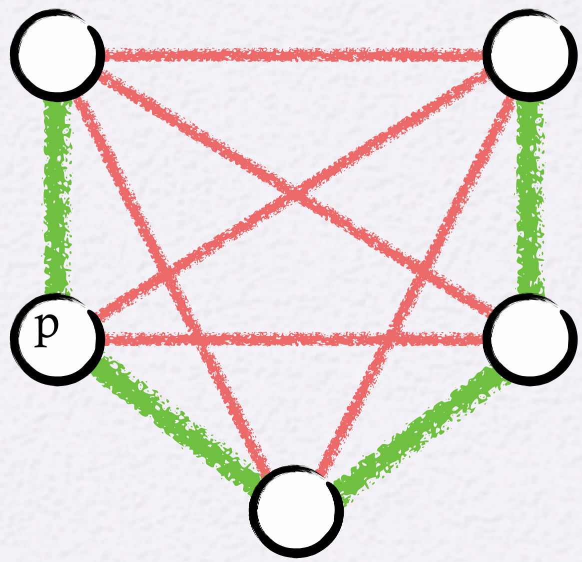
- Pick a node p uniformly at random
- Create a cluster around p by including all nodes connected to p by a +1 edge
- Delete the nodes in this cluster
- Repeat with the remaining graph

[Ailon, Charikar, Newman]

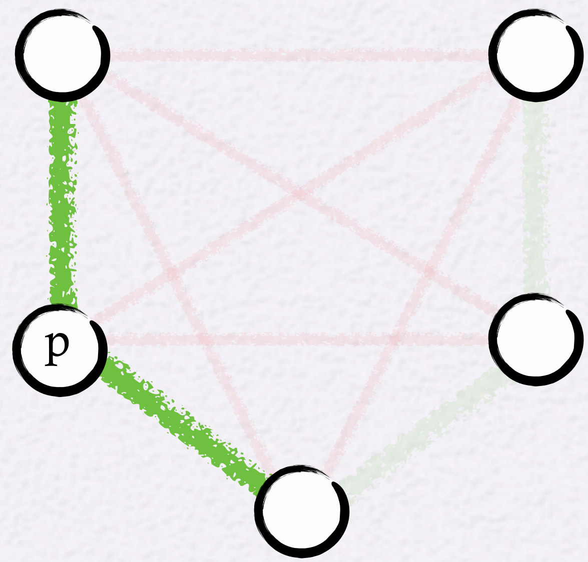
Eg: Pivot Algorithm



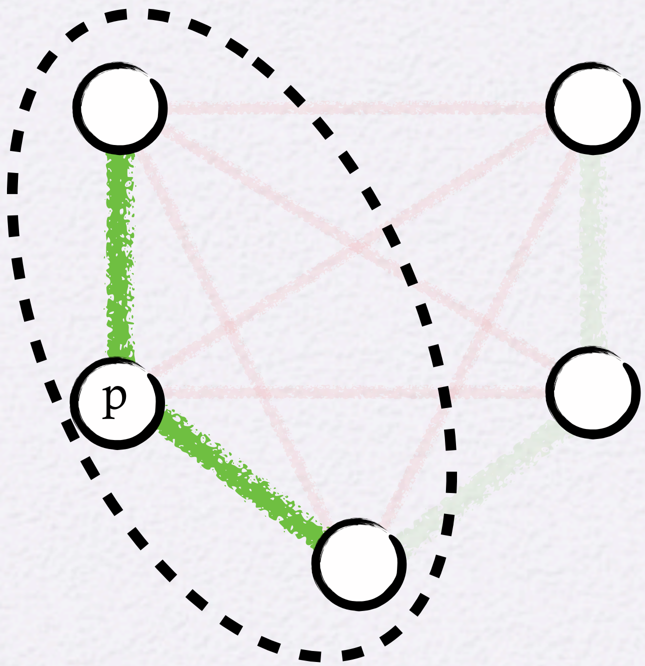
Eg: Pivot Algorithm



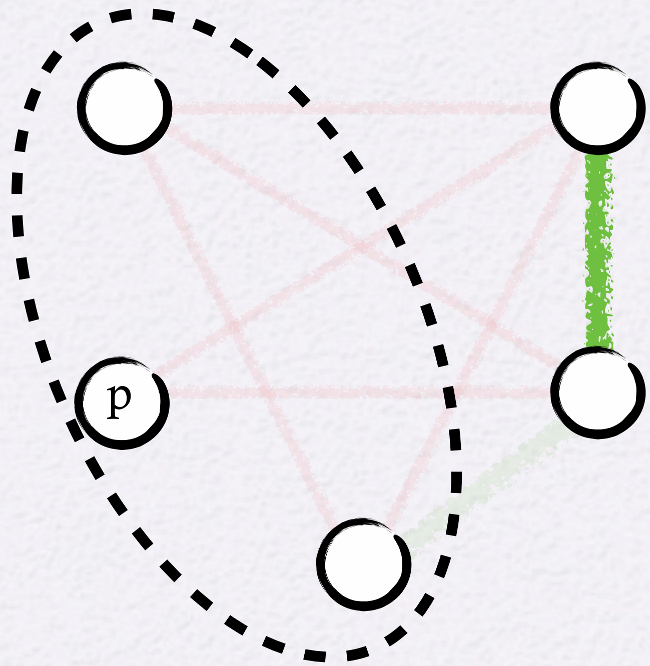
Eg: Pivot Algorithm



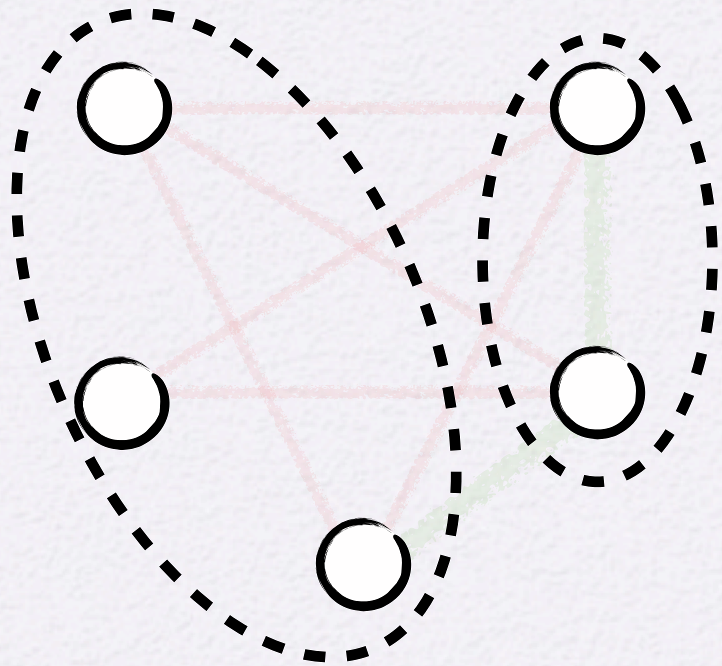
Eg: Pivot Algorithm



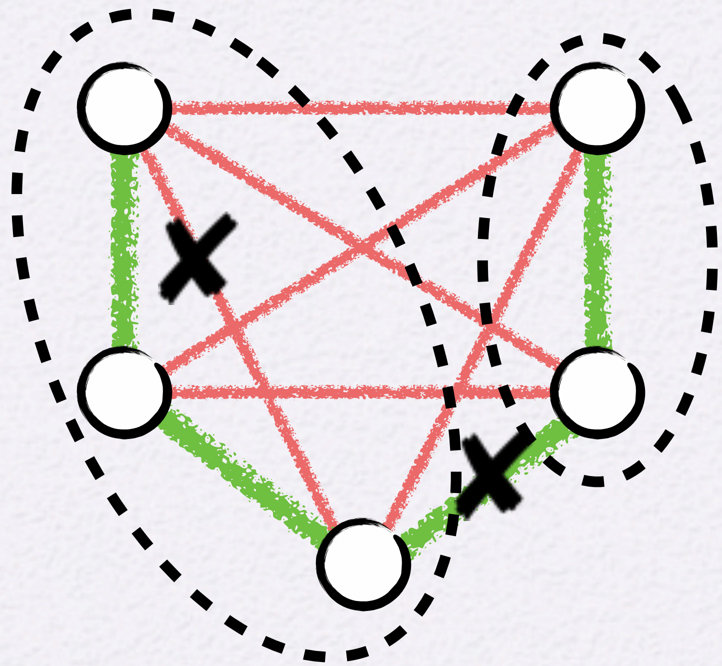
Eg: Pivot Algorithm



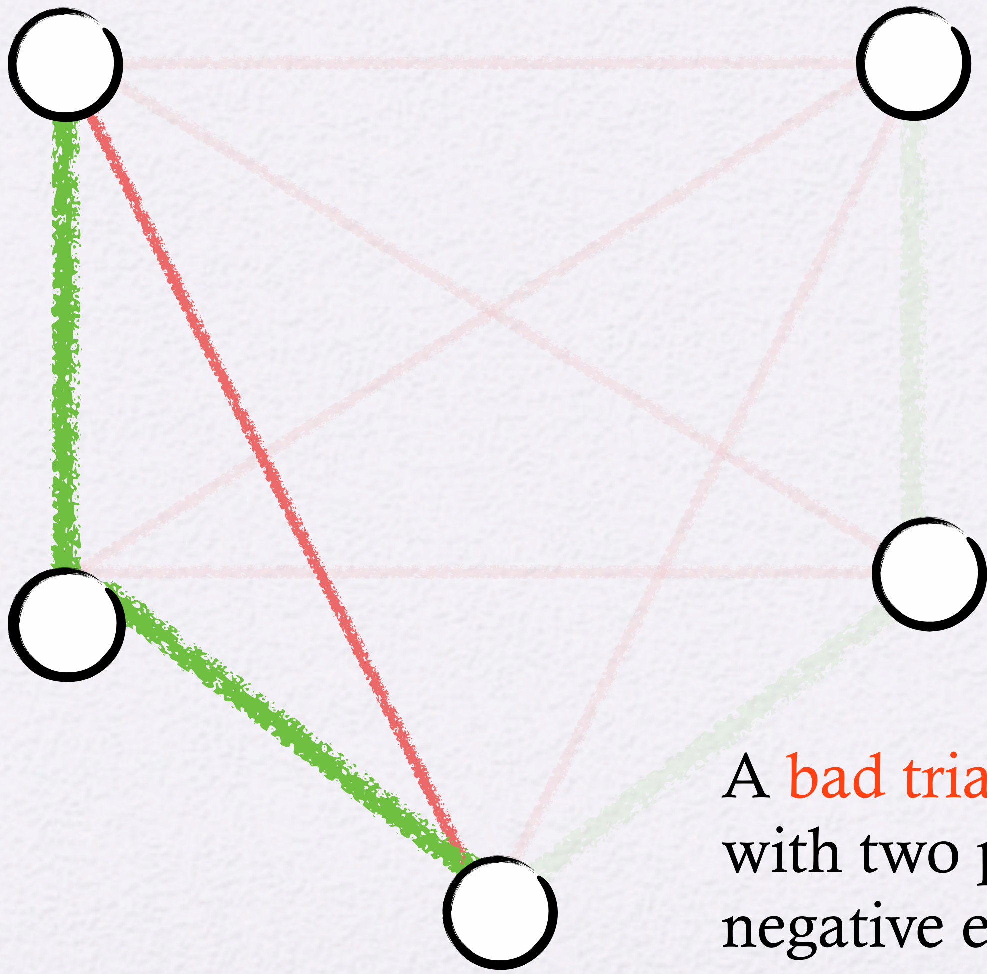
Eg: Pivot Algorithm



Eg: Pivot Algorithm



Bad triangles



A bad triangle is a triple of nodes with two positive edges and one negative edge

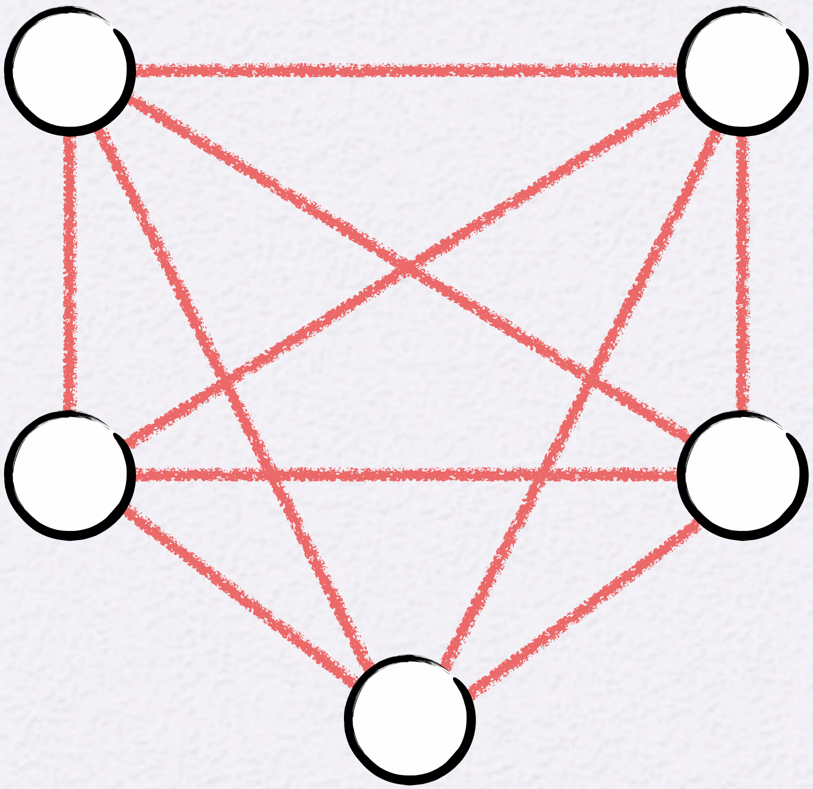
Properties

Claim [ACN]. Pivot gives (in expectation) a 3-approximation to minimizing the number of mistakes

Proof focuses on bad triangles and uses LP duality

The algorithm is inherently sequential

A bad example



Pivot takes $\Omega(n)$ rounds

Parallel Pivot

- A parallel version of Pivot Algorithm
  - runs in $O(\log^2 n)$ rounds
  - obtains a $3+\varepsilon$ approximation

[Chierichetti, Dalvi, Kumar]

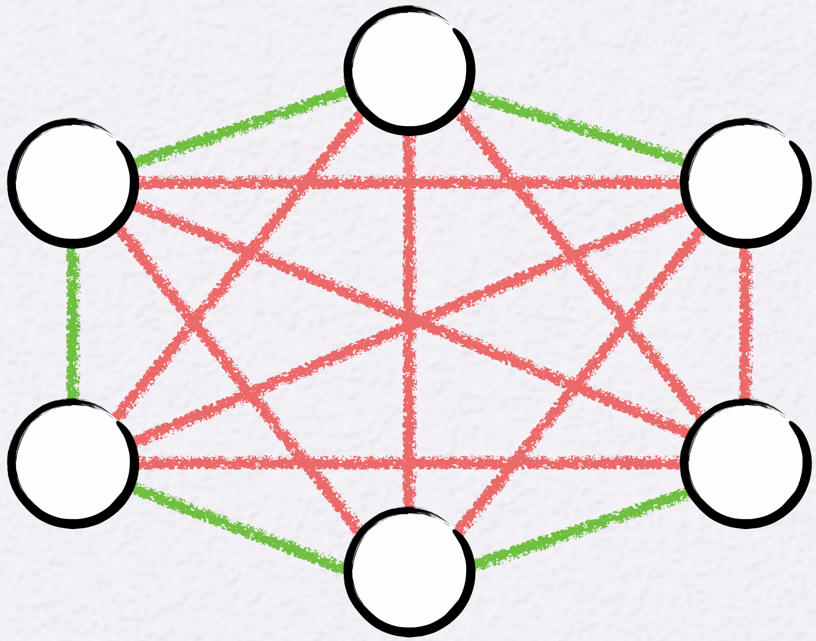
- Easily implemented in streaming (also Map-Reduce, Pregel, …)

Parallel Pivot Algorithm

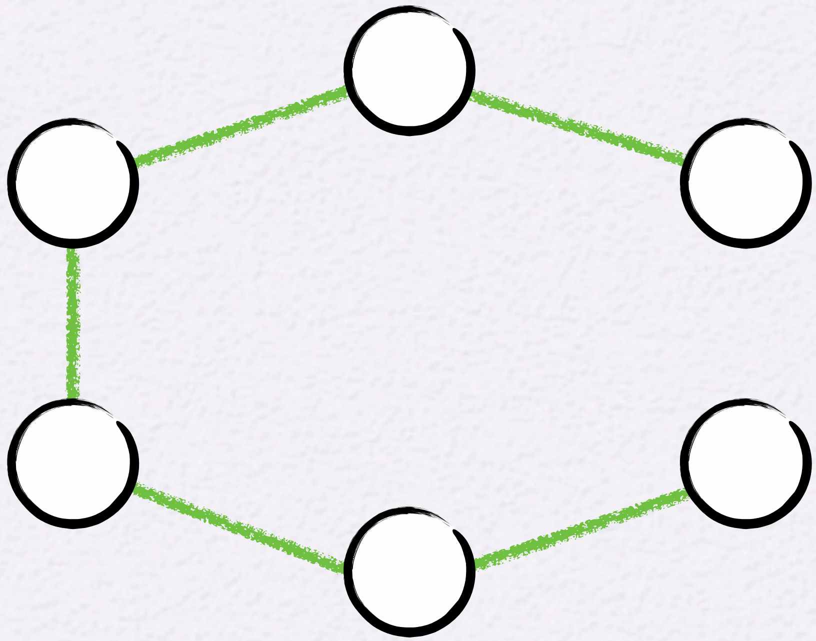
While the graph is not empty

- Let D^+ be the current maximum positive degree
- Activate each node independently wp ε/D^+
- Deactivate nodes connected to other active nodes by +1 edges
- The remaining nodes are pivots
- Create cluster around each pivot as before
- Remove the clusters

Example

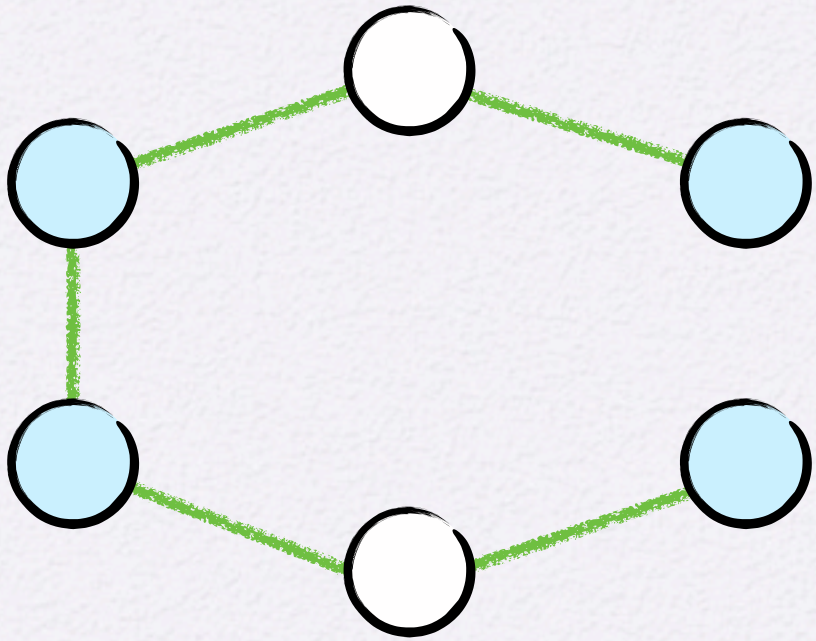


Example

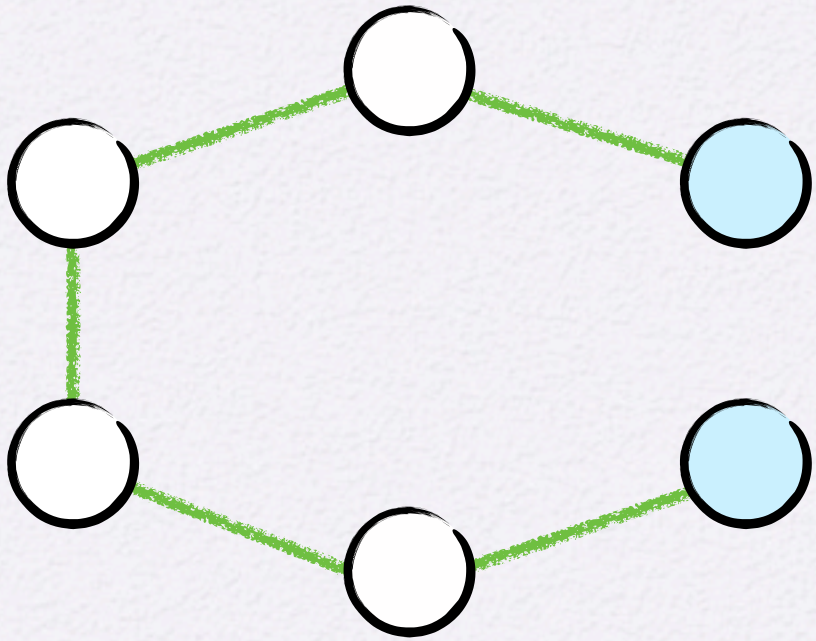


$D^+ = 2$

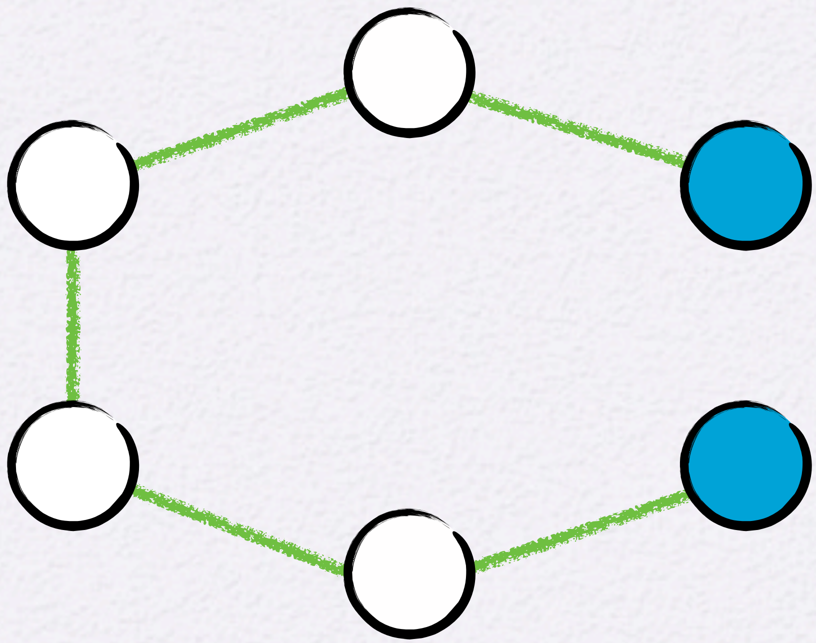
Example



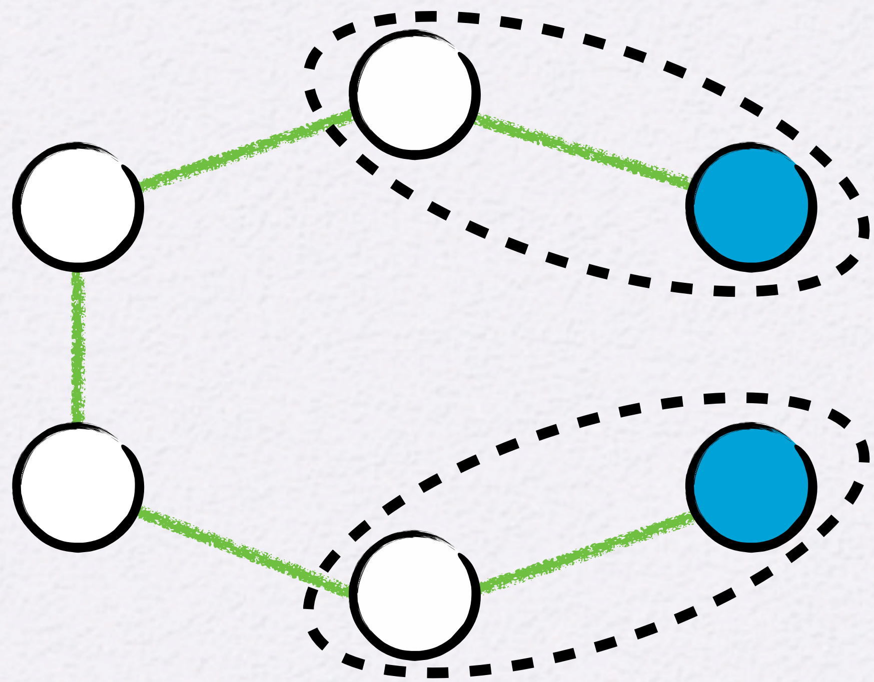
Example



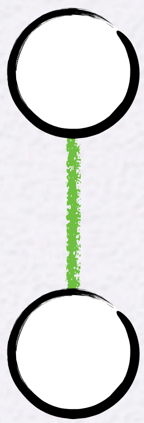
Example



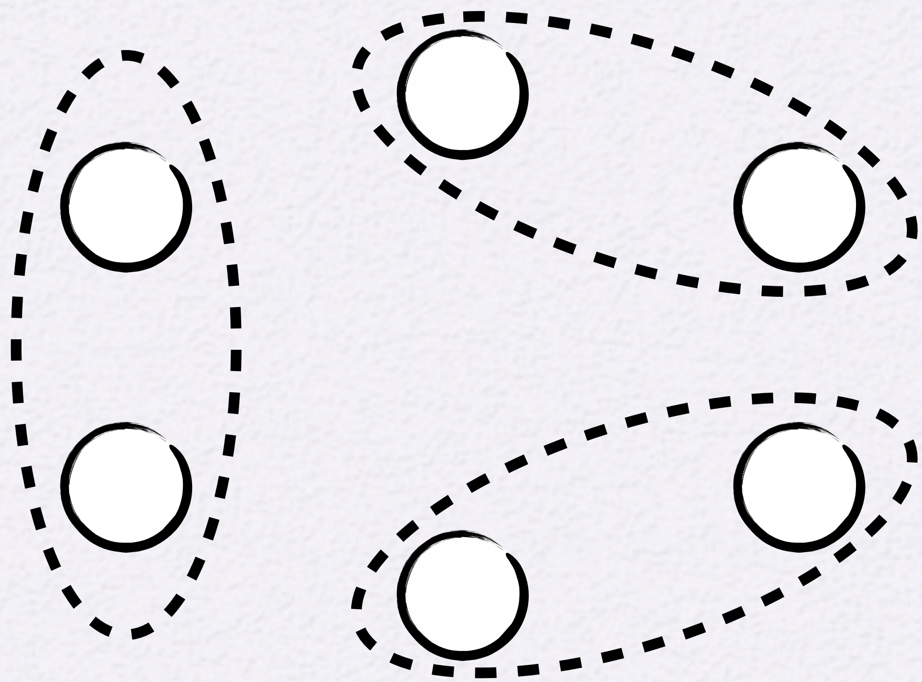
Example



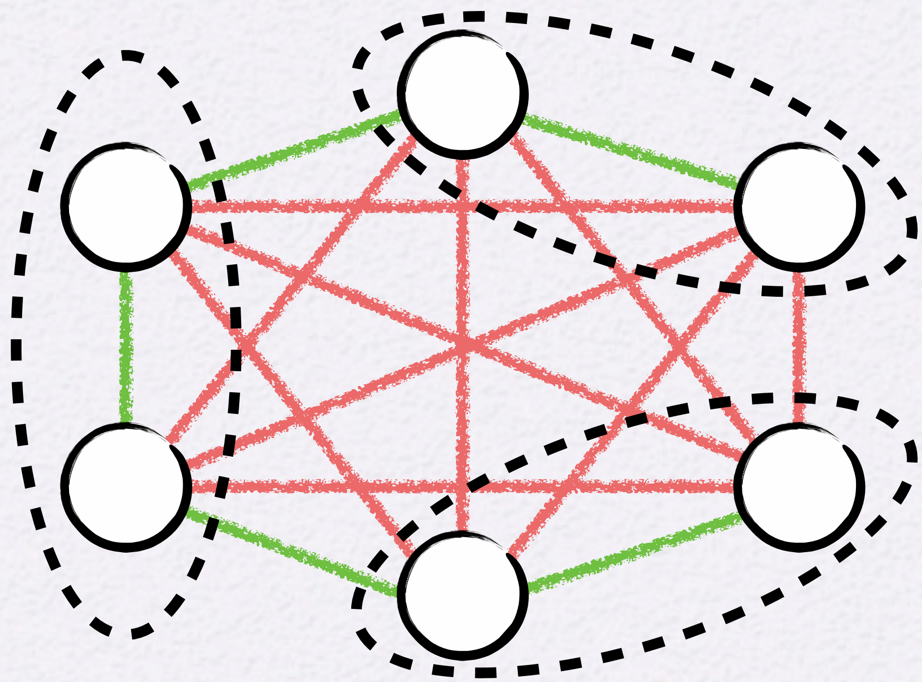
Example



Example



Example



Properties

Claim. Parallel Pivot halves the maximum degree D^+ after $(1/\varepsilon)$ log n rounds

Algorithm terminates in $(1/\varepsilon)$ (log n) (log D^+) rounds

Claim. Induces a close to uniform marginal distribution of the pivots

Can extend the LP dual-based proof of [ACN] to show $3 + \varepsilon$ approximation

Halving max degree

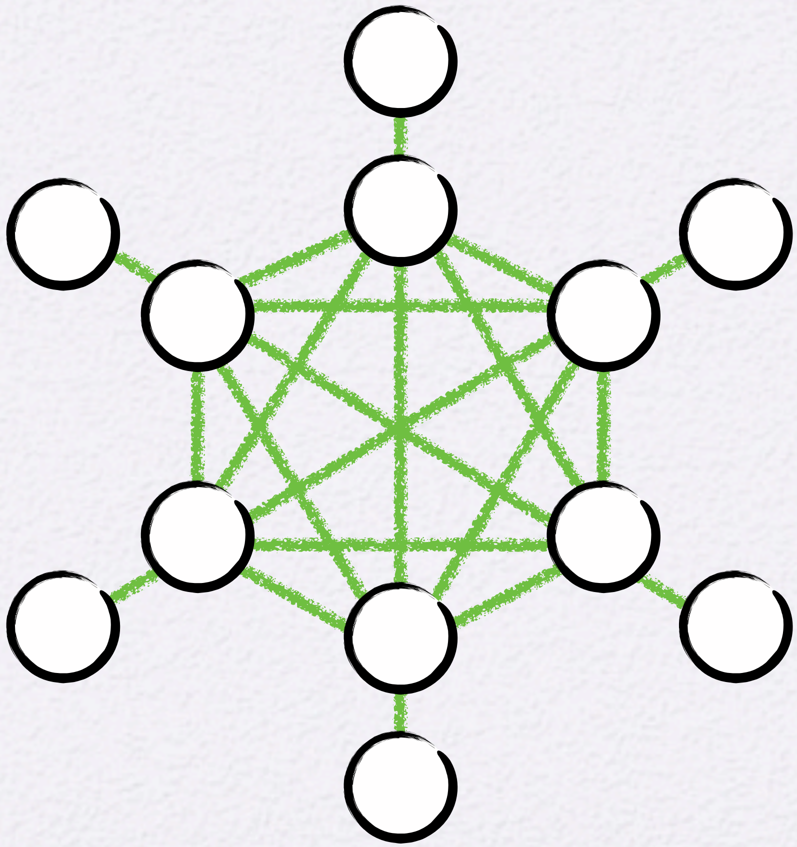
- Event e(v): exactly one positive neighbor w of node v gets activated and no positive neighbor of w gets activated
  - w becomes a pivot and hence v is removed
- Key property: $\Pr[e(v)] > \varepsilon/8$ if $\deg^+(v) > D^+/2$
- After logarithmic number of rounds, either v's positive degree halves or v will end up in a cluster

Different sampling?

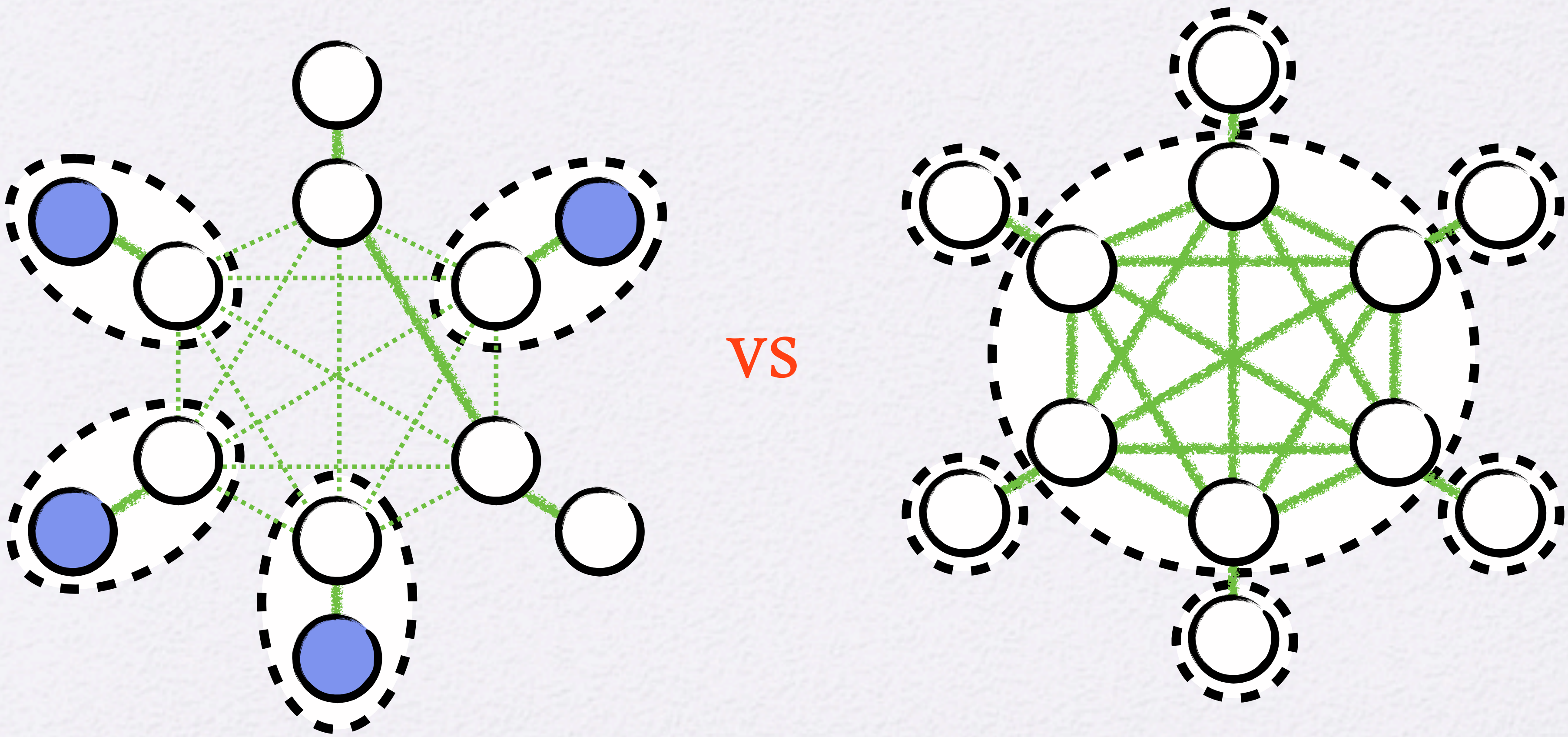
Other natural sampling methods can produce a non-constant approximation

- node u is activated wp deg(u)
  - Eg, star of degree n
- node u is activated wp 1/deg(u) [Luby]
  - Eg, a clique matched to an independent set of nodes

Eg. inverse degree



Algorithm vs Optimum

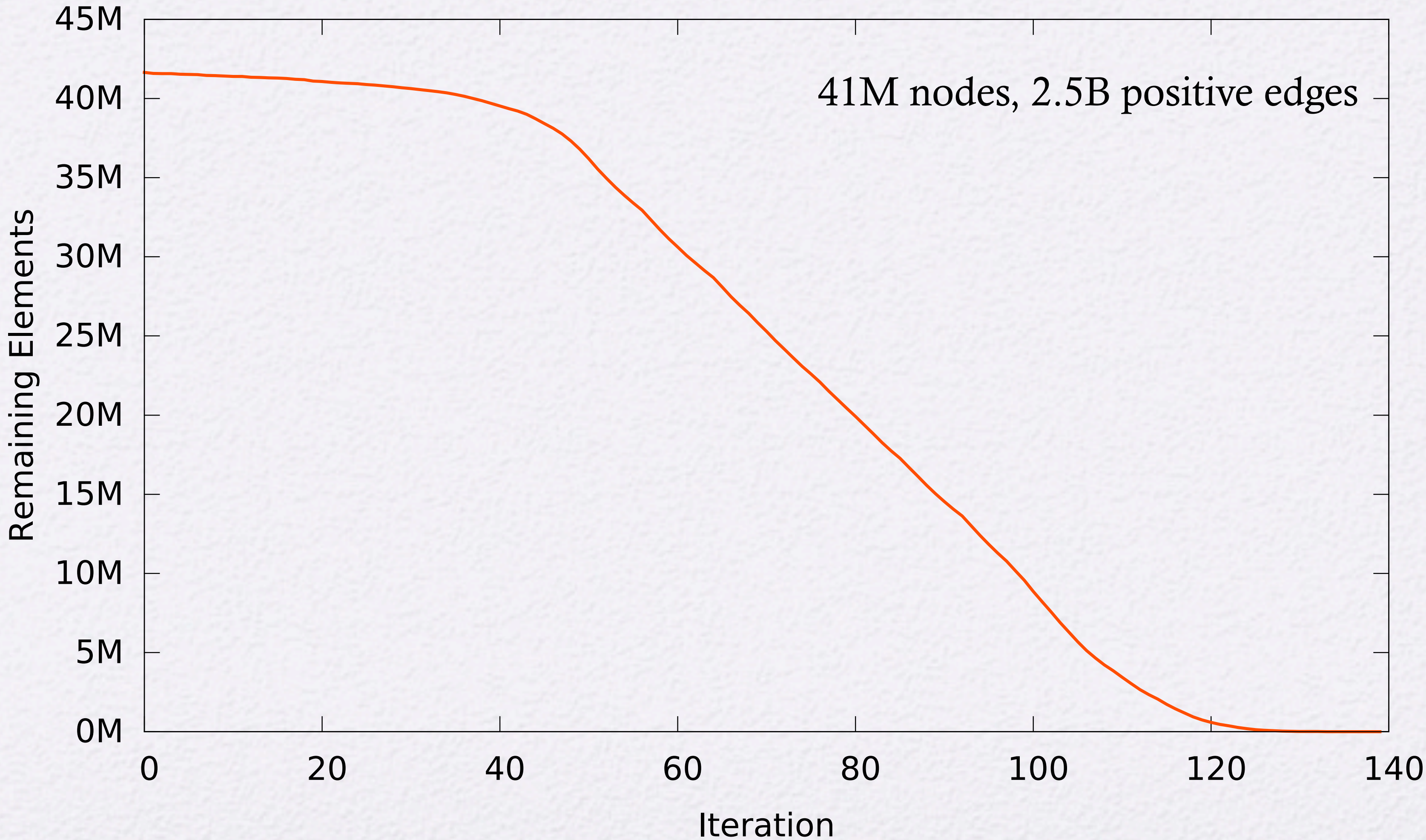


Different sampling?

Other uniform sampling approaches might require more rounds

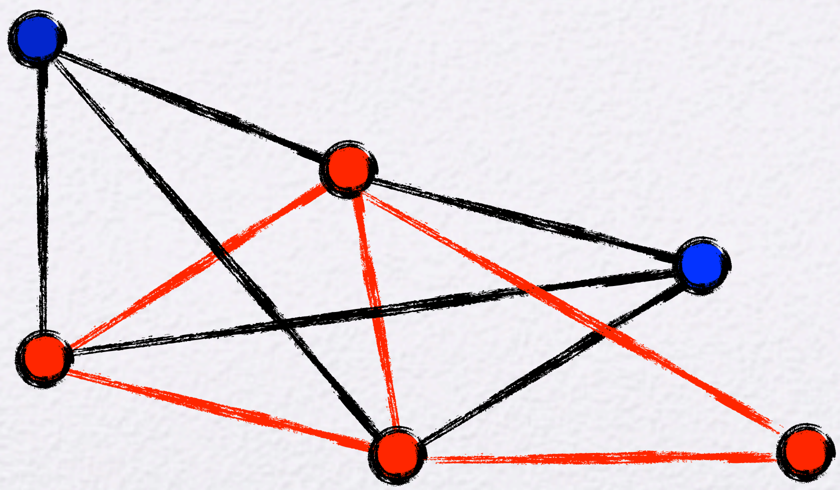
- node u is activated wp << $1/D^+$
  - few active nodes, few pivots, many rounds
- node u is activated wp >> $1/D^+$
  - many active nodes, few pivots, many rounds
  - pivots far from uniform distribution

Twitter Dataset



2. Densest subgraph (DSG)

- Find densest subgraph in undirected graphs
  - Density of a subgraph is the ratio of the number of edges to the number of nodes
  - Motivation: Community finding
  - c-approximation = when density is at most c times worse then the best density



Density(●) = 5/4 = 1.2

Complexity of DSG

- DSG can be computed in polynomial time
  - Using parametric flows or LP relaxation
- Natural variants of DSG are hard
  - k-DSG, subgraph with exactly k nodes
- Charikar's 2-approximation algorithm
  - Iteratively remove the lowest degree node until the graph becomes empty
  - One of the intermediate graphs is a 2-approx.
- These algorithms are hard to scale

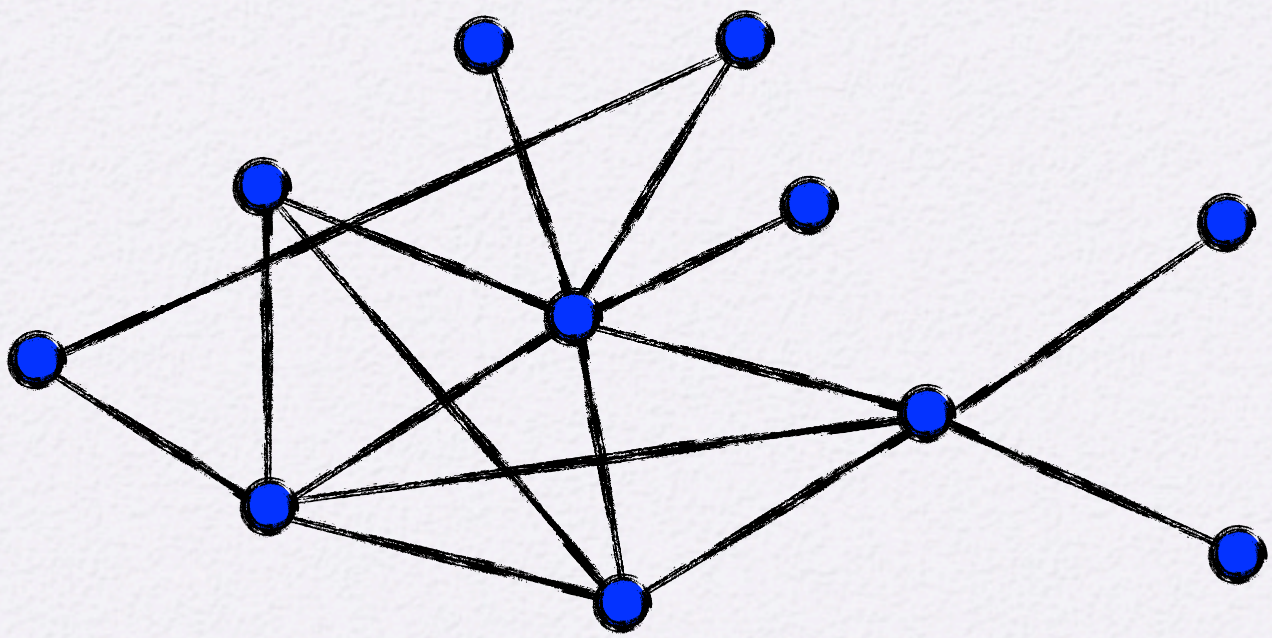
DSG: Algorithm

A simple iterative algorithm

- Compute the average degree
- Delete all nodes whose degree is $(1+\varepsilon)$ below the average
- Keep track of the density at each step
- Output the densest graph seen during the iteration

[Bahmani, Kumar, Vassilvitskii]

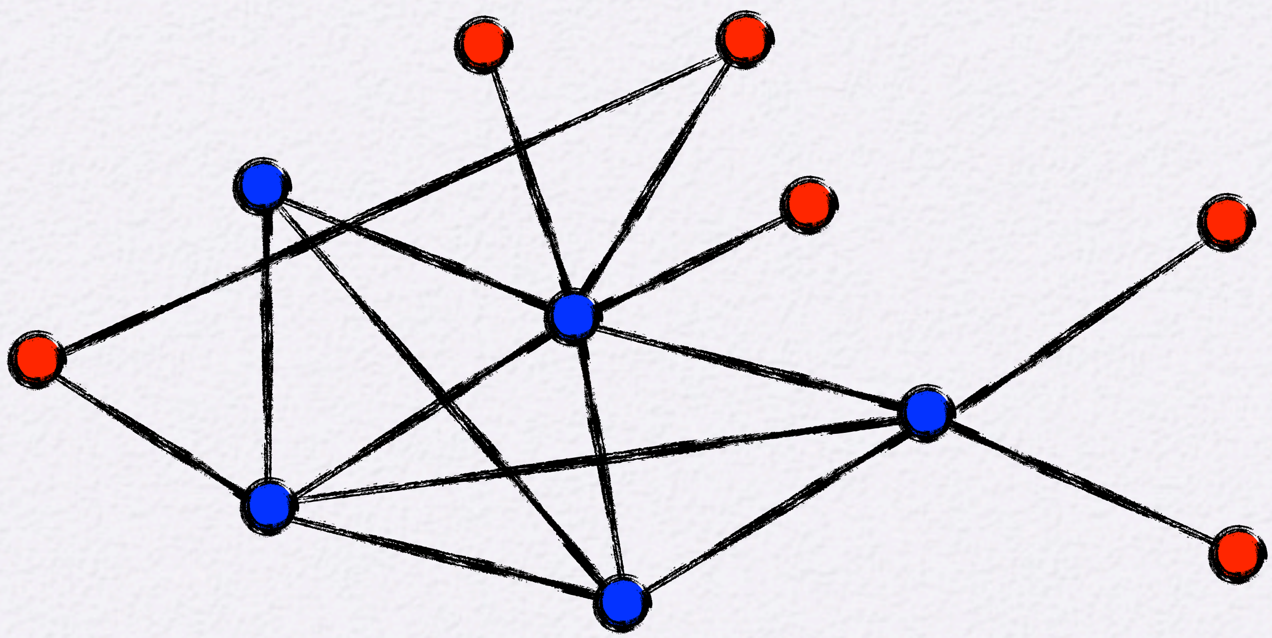
DSG: Example



density = 16/11 = 1.45; average degree = 2*density = 2.90

Best density = 1.45

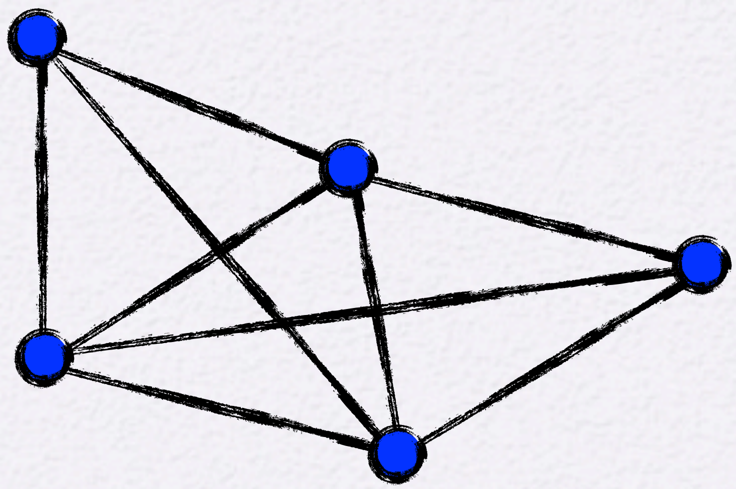
DSG: Example (contd)



density = 16/11 = 1.45; average degree = 2*density = 2.90

Best density = 1.45

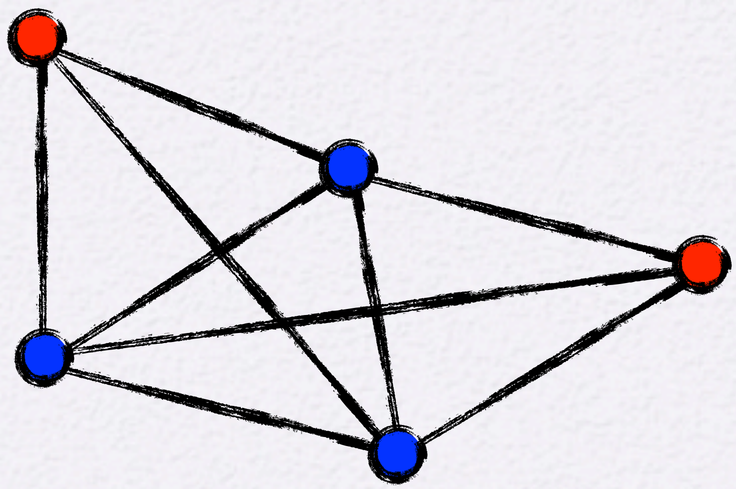
DSG: Example (contd)



density = 9/5 = 1.8; average degree = 2*density = 3.6

Best density = 1.8

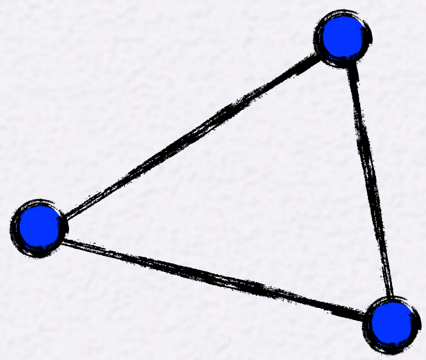
DSG: Example (contd)



density = 9/5 = 1.8; average degree = 2*density = 3.6

Best density = 1.8

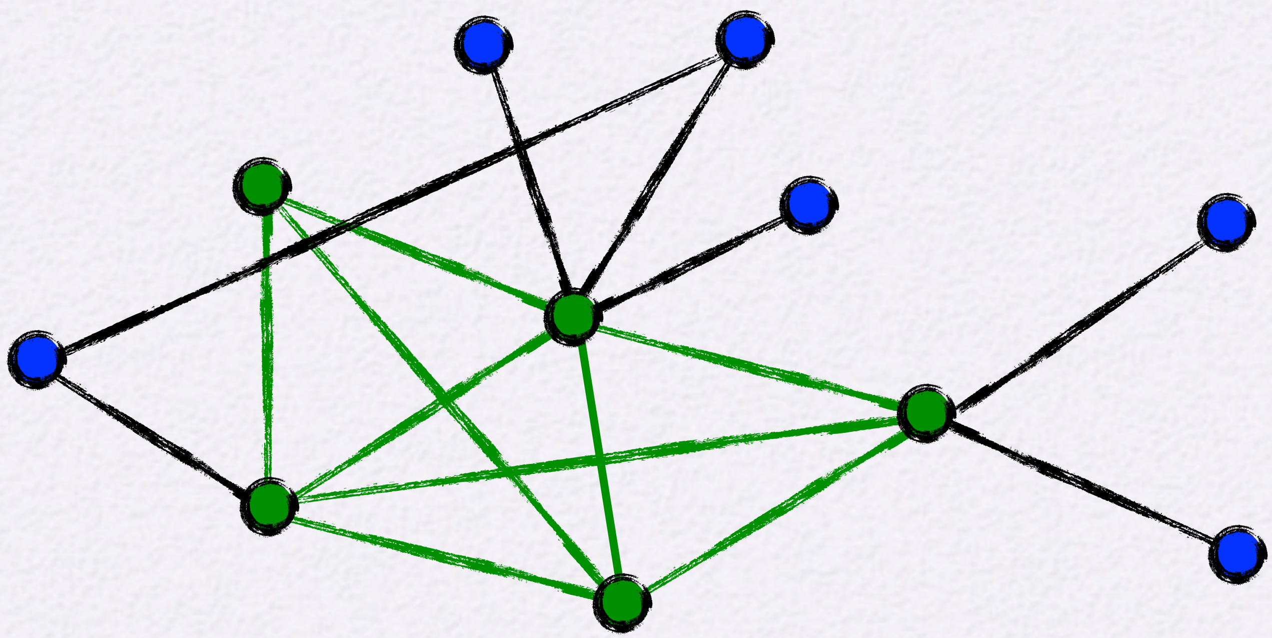
DSG: Example (contd)



density = 3/3 = 1; average degree = 2*density = 2

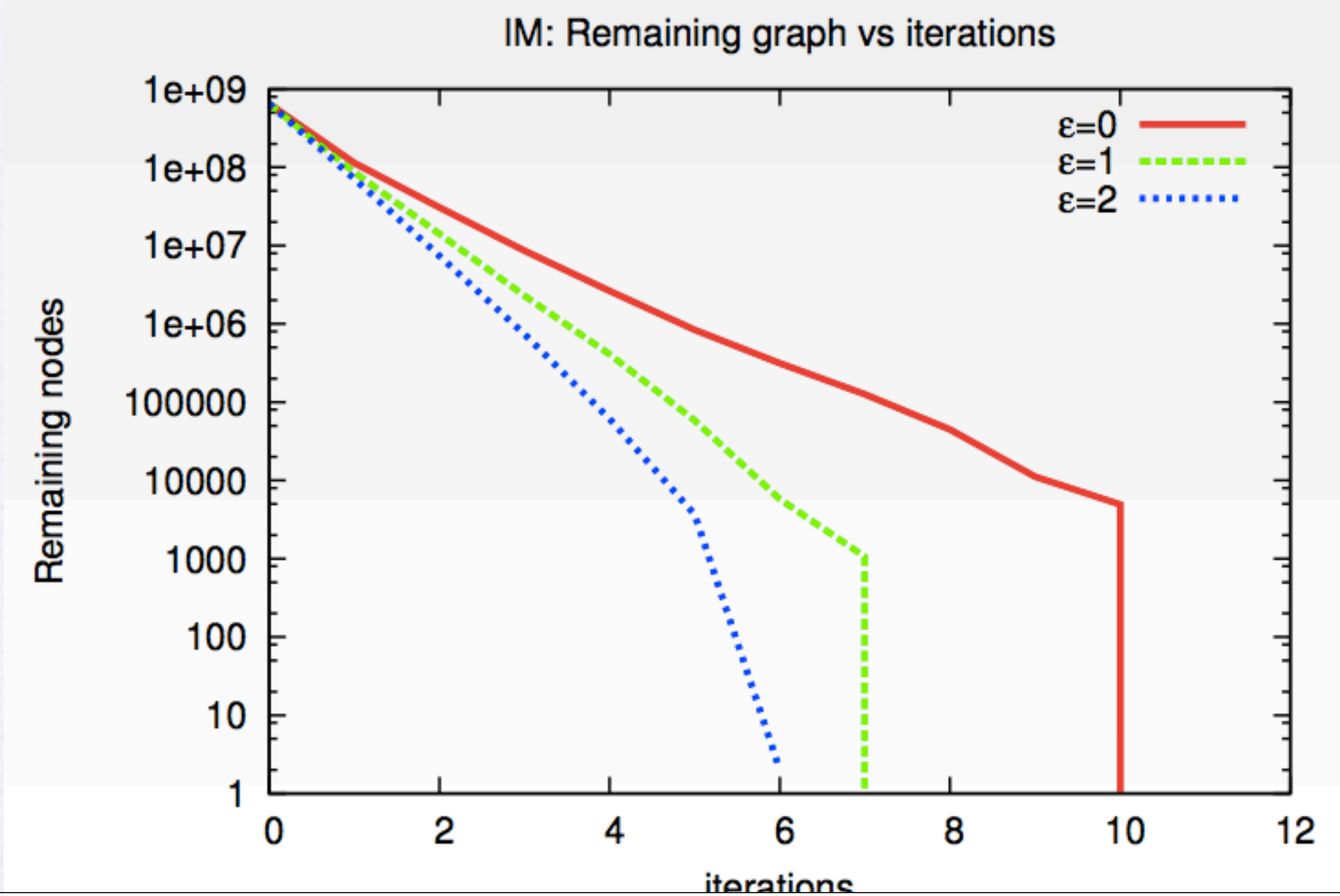
Best density = 1.8

DSG: Example (contd)



Best density = 1.8

DSG: Performance



Properties

Claim. Algorithm makes $O(\log_{1+\varepsilon} n)$ passes and uses $O(n)$ memory

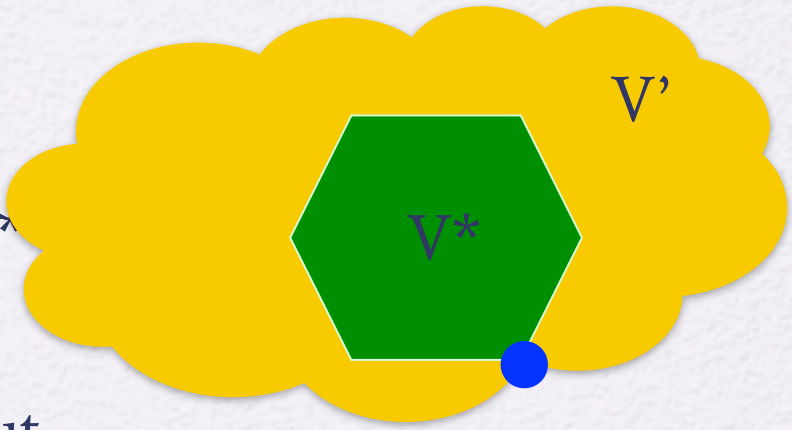
Use an averaging argument

Claim. Output is a $(2+\varepsilon)$-approx.

V^* = optimal induced subgraph, p^* = density(V^*)

Each node in V^* has degree at least p^* (optimality)

V' = first subgraph where we are about to remove a node in V^*



Concluding thoughts

- Non-traditional computational models are key to managing big graphs
  - Novel algorithmic ideas
  - New programming paradigms
- Round complexity is important
  - One-pass 2-approximation algorithm for DSG [Bhattacharya, Henzinger, Nanongkai, Tsourakakis]
  - Correlation clustering?
  - k-means^{++}?
- Managing heavy tail, data skew, asynchrony, communication, …

Thank you!

Questions/Comments

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