Massively Parallel Communication and Query Evaluation

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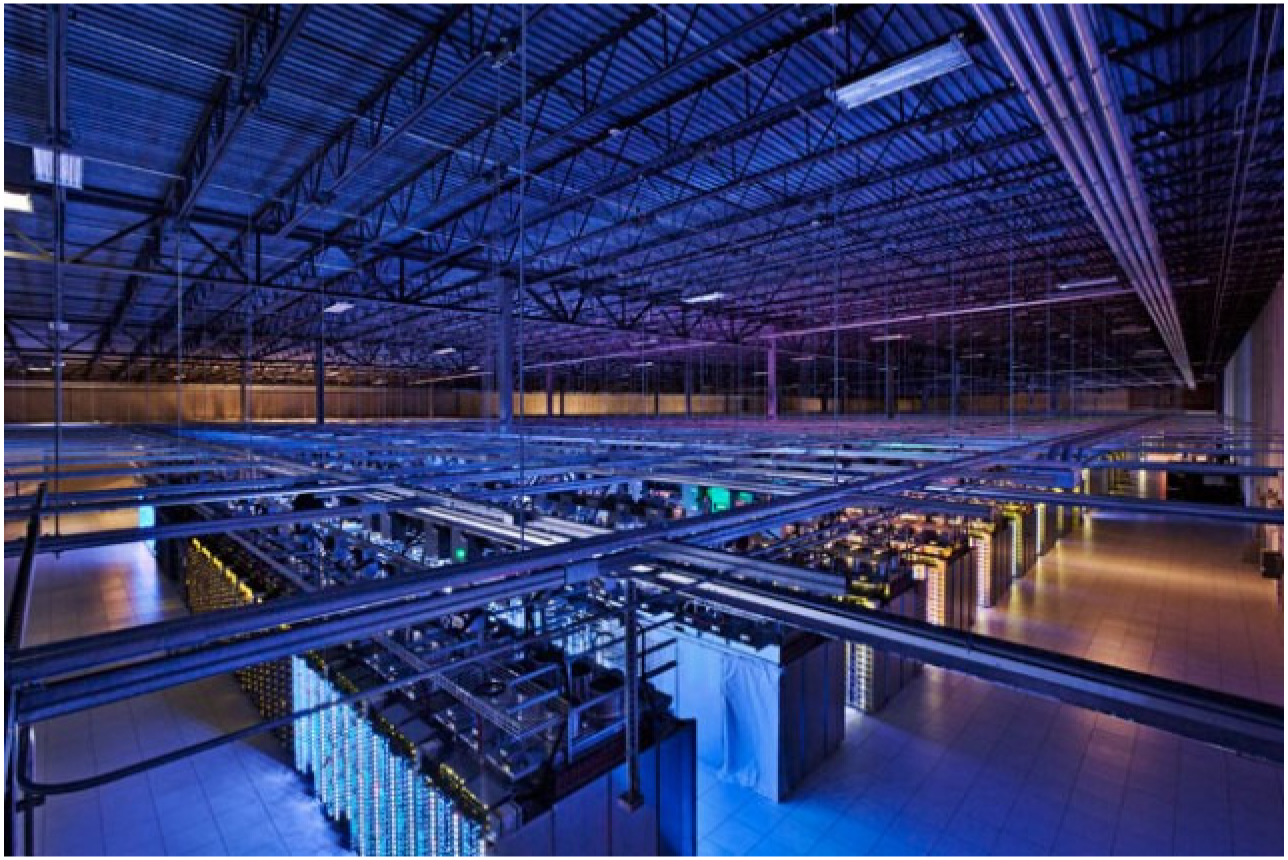
Based on joint work with Paraschos Koutris and Dan Suciu
[PODS 13], [PODS 14]

Massively Parallel Systems



MapReduce

[Dean,Ghemawat 2004] Rounds of

Map: Local and data parallel on *(key, value)* pairs creating $(key_1,value_1)$… $(key_k,value_k)$ pairs

Shuffle: Groups or sorts *(key, value)* pairs by key

- Local sorting plus global communication round

Reduce: Local and data parallel on *keys*: $(key,value_1)$… $(key,value_k)$ reduces to *(key,value)*

– Data fits jointly in main memory of 100's/1000's of parallel servers each with gigabyte$^+$ storage

– Fault tolerance

What can we do with MapReduce? Models & Algorithms

- Massive Unordered Distributed Data (MUD) model [Feldman-Muthukrishnan-Sidiropoulos-Stein-Svitkina 2008]
  - 1 round can simulate data streams on symmetric functions, using Savitch-like small space simulation
  - Exact computation of frequency moments in 2 rounds of MapReduce
- $\mathcal{MRC}$ model [Karloff, Suri, Vassilvitskii 2010]
  - For $\mathbf{n^{1-\varepsilon}}$ processors and $\mathbf{n^{1-\varepsilon}}$ storage per processor, $\mathbf{O(t)}$ rounds can simulate $\mathbf{t}$ PRAM steps so $\mathbf{O}(\log^k \mathbf{n})$ rounds can simulate NC^k
  - Minimum spanning trees and connectivity *on dense graphs* in 2 rounds of MapReduce
  - Generalization of parameters, sharper simulations, sorting and computational geometry applications [Goodrich, Sitchinava, Zhang 2011]

What can we do with MapReduce? Models & Algorithms

- Communication-processor tradeoffs for 1 round of MapReduce
  - Upper bounds for database join queries [Afrati,Ullman 2010]
  - Upper and lower bounds for finding triangles, matrix multiplication, finding neighboring strings [Afrati, Sarma, Salihoglu, Ullman 2012]

More than just MapReduce

What can we do with this?

Are there limits? Lower bounds?

A simple general model?

MapReduce

$\Omega(n \log n)$ time

[Dean,Ghemawat 2004] Rounds of

- Map: Local and data parallel on *(key, value)* pairs creating *(key$_1$,value$_1$)... (key$_k$,value$_k$)* pairs
- Shuffle: Groups or sorts *(key, value)* pairs by key
  - Local sorting plus global communication round
- Reduce: Local and data parallel on *keys*: *(key,value$_1$)... (key,value$_m$)* reduces to *(key,value)*

unspecified time

– Data fits jointly in main memory of 100's/1000's of parallel servers each with gigabyte$^+$ storage

– Fault tolerance

essential for efficiency

Properties of a Simple General Model of Massively Parallel Computation

- Organized in synchronous rounds
- Local computation costs per round should be considered free, or nearly so
  - No reason to assume that sorting is special compared to other operations
- Memory per processor is the fundamental constraint
  - This also limits # of bits a processor can send or receive in a single round

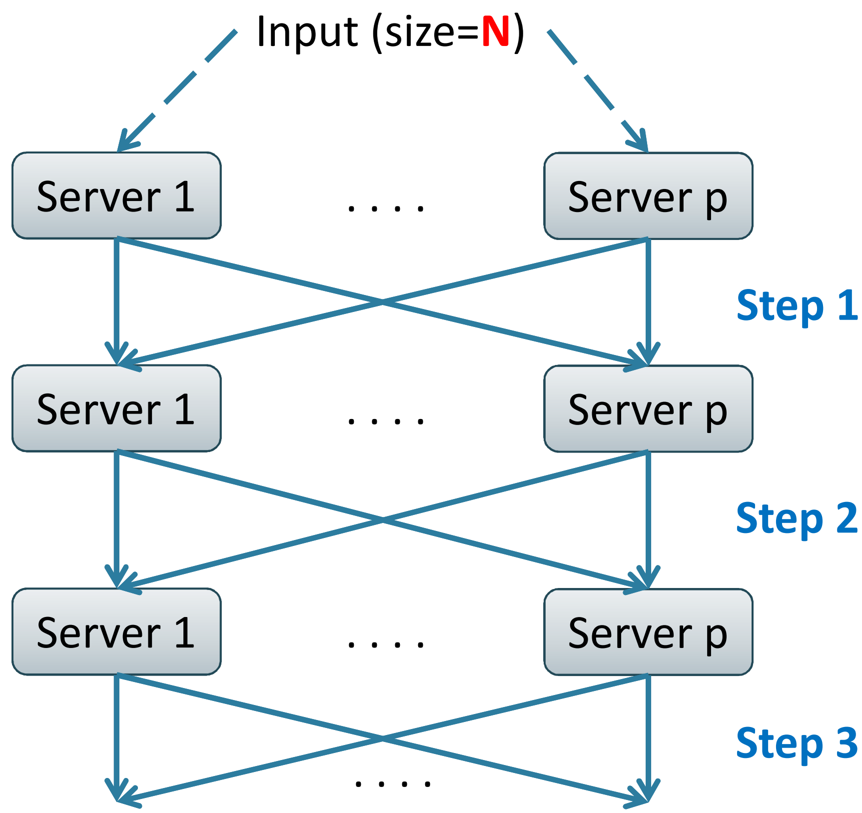
Bulk Synchronous Parallel Model

[Valiant 1990] Local computations separated by global synchronization barriers

- Key notion: An ***h****-relation,* in which each processor sends and receives at most ***h*** bits
- Parameters:
  - periodicity ***L*** : time interval between synchronization barriers
  - bandwidth ***g*** : $max_{\boldsymbol{h} \geq \boldsymbol{h_0}} \frac{time\ to\ deliver\ an\ \boldsymbol{h}-relation}{\boldsymbol{h}}$

Massively Parallel Communication (MPC) Model

- Total size of the input = **N**
- Number of processors = **p**
- Each processor has:
  - Unlimited computation power
  - **L** ≥ **N/p** bits of memory
- A round/step consists of:
  - Local computation
  - Global communication of an **L**-relation
    - i.e., each processor sends/receives ≤ **L** bits
    - **L** stands for the communication/memory *load*



MPC model continued

- Wlog $\mathbf{N/p} \leq \mathbf{L} \leq \mathbf{N}$
  - any processor with access to the whole input can compute any function
- Communication
  - processors pay *individually for receiving* the **L** bits per round, total communication cost up to $\mathbf{pL} \geq \mathbf{N}$ per round.
- Input distributed uniformly
  - Adversarially or random input distribution also
- Access to random bits (possibly shared)

Relation to other communication models

- Message-passing (private messages) model
  - each costs per processor receiving it
  - wlog one player is a Coordinator who sends and receives every message
    - Many recent results improving **Ω(N/p)** lower bounds to **Ω(N)** [WZ 12], [PVZ12], WZ13], [BEOPV13],...
    - Complexity is never larger than **N** bits independent of rounds
  - No limit on bits per processor, unlike MPC model
- $\mathcal{CONGEST}$ model
  - Total communication bounds > **N** possible but depends on network diameter and topology
    - MPC corresponds to a complete graph for which largest communication bound possible is $\leq$ **N**

Complexity in the MPC model

- Tradeoffs between rounds **r**, processors **p**, and load **L**

- Try to minimize load **L** for each fixed **r** and **p**
  - Since **N/p** $\leq$ **L** $\leq$ **N**, the range of variation in **L** is a factor **p^{ε}** for **0≤ε≤1**

- 1 round
  - still interesting theoretical/practical questions
  - many open questions

- Multi-round computation more difficult
  - e.g. PointerJumping, i.e., st-connectivity in out-degree 1 graphs.
    - Can achieve load **O(N/p)** in **r**=O($\log_2$**p**) rounds by pointer doubling

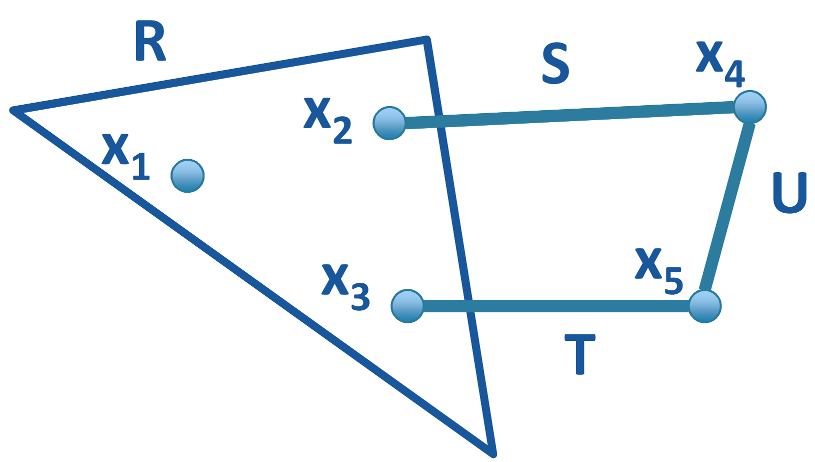
Database Join Queries

- Given input relations $\mathbf{R_1, R_2, \ldots, R_m}$ as tables of tuples, of possibly different arities, produce the table of all tuples answering the query $\mathbf{Q(x_1, x_2, \ldots, x_k) = R_1(x_1,x_2,x_3), R_2(x_2,x_4),\ldots, R_m(x_4,x_k)}$
  - Known as full conjunctive queries since every variable in the RHS appears in the query (no variables projected out)
- Our examples: Connected queries only

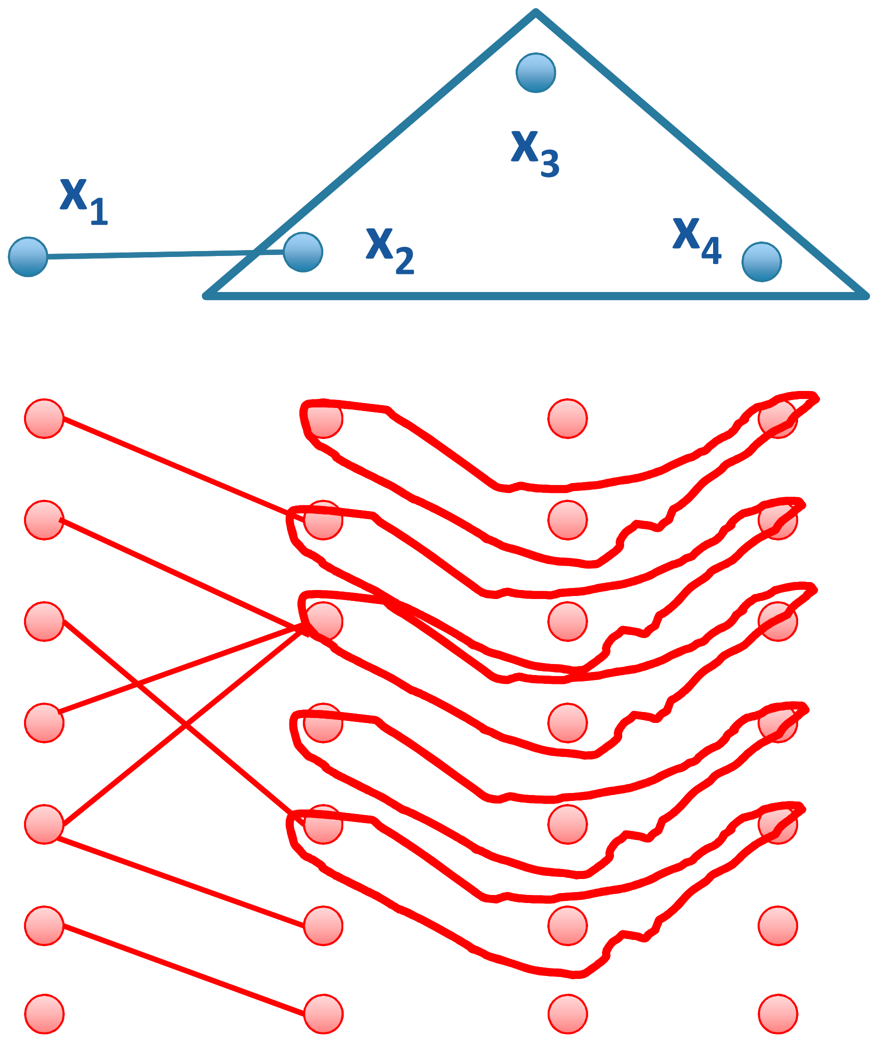
The Query Hypergraph

- One vertex per variable
- One hyper-edge per relation

$$Q(x_1, x_2, x_3, x_4, x_5) = R(x_1,x_2,x_3), S(x_2,x_4), T(x_3,x_5), U(x_4,x_5)$$



k-partite data graph/hypergraph



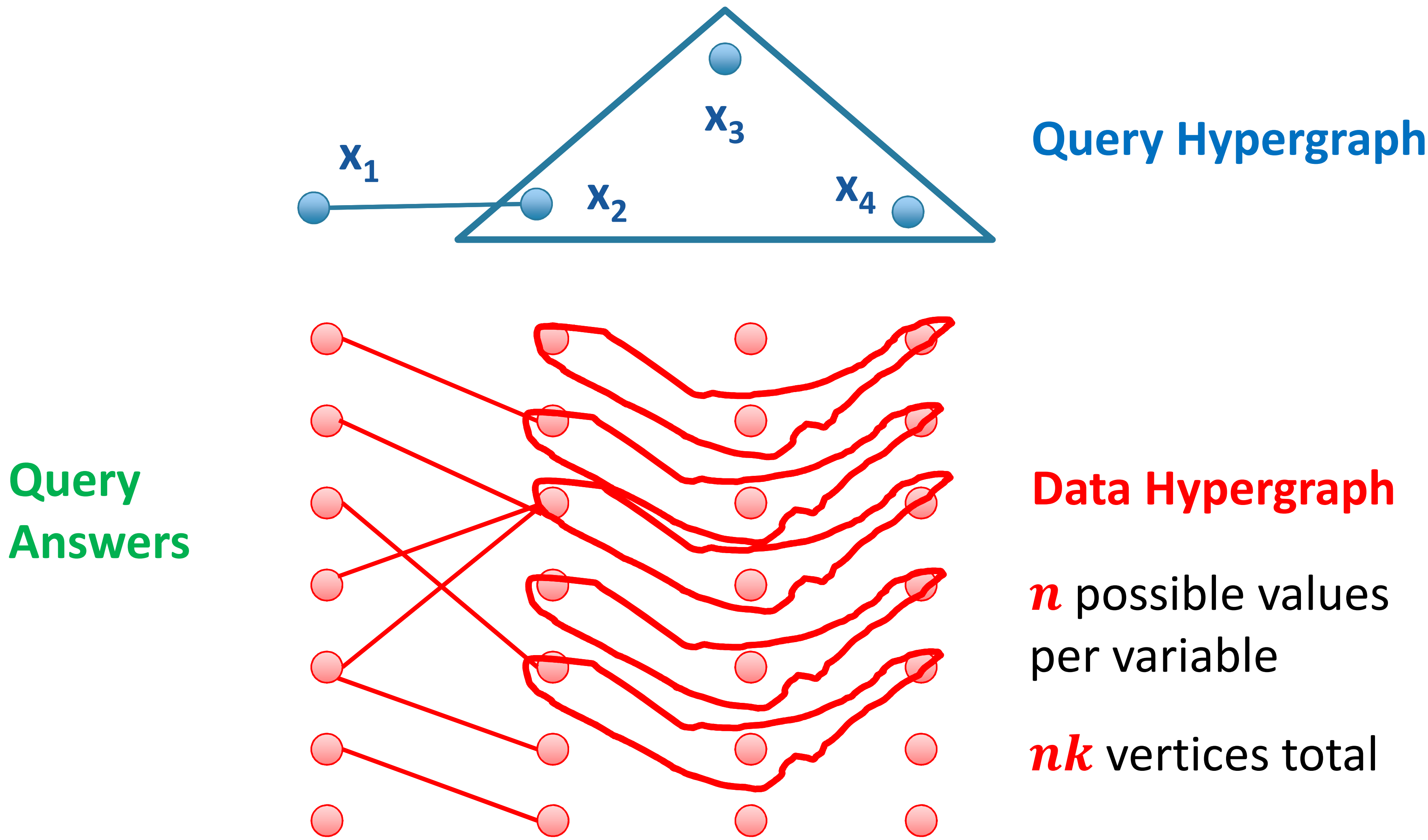
Query Hypergraph

Data Hypergraph

n possible values per variable

nk vertices total

k-partite data graph/hypergraph



n possible values per variable

nk vertices total

Some Hard Inputs

- Matching Databases
  - Number of relations $R_1, R_2, \ldots$ and size of query is constant
  - Each R_j is a perfect a_j-dimensional **matching** on $[n]^{a_j}$ where a_j is the arity of R_j
    - i.e. among all the a_j-tuples $(k_1,\ldots,k_{a_j}) \in R_j$, each value $k \in [n]$ appears exactly once in each coordinate.
    - No skew (all degrees are the same)
    - Number of output tuples is at most n
  - Total input size is $N = O(\log(n!)) = O(n \log n)$

Example in two steps

Find all triangles

$\mathbf{C_3(x,y,z) = R_1(x,y), R_2(y,z), R_3(z,x)}$

$\mathbf{R_1}$=

X	Y
a1	b3
a2	b1
a3	b2

$\mathbf{R_2}$=

Y	Z
b1	c2
b2	c3
b3	c1

$\mathbf{R_3}$=

Z	X
c1	a2
c2	a1
c3	a3

$\mathbf{C_3}$ =

X	Y	Z
a3	b2	c3

Algorithm 1:

For each server $\mathbf{1 \le u \le p}$:

Input: $\mathbf{n/p}$ tuples from each of $\mathbf{R_1, R_2, R_3}$

Step 1: send $\mathbf{R_1(x,y)}$ to server ($\mathbf{y}$ mod $\mathbf{p}$)
send $\mathbf{R_2(y,z)}$ to server ($\mathbf{y}$ mod $\mathbf{p}$)

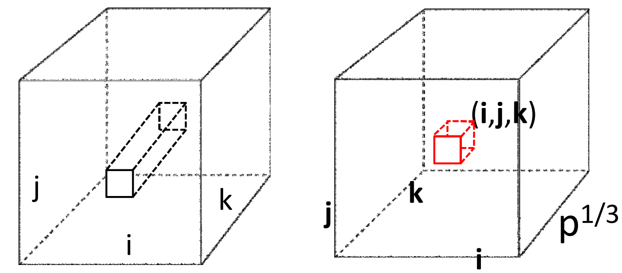
Step 2: join $\mathbf{R_1(x,y)}$ with $\mathbf{R_2(y,z)}$
send $\mathbf{[R_1(x,y),R_2(y,z)]}$ to server ($\mathbf{z}$ mod $\mathbf{p}$)
send $\mathbf{R_3(z,x)}$ to server ($\mathbf{z}$ mod $\mathbf{p}$)

Output join $\mathbf{[R_1(x,y),R_2(y,z)]}$ with $\mathbf{R_3(z,x')}$
output all triangles $\mathbf{R_1(x,y),R_2(y,z),R_3(z,x)}$

Load: $\mathbf{O(n/p)}$ tuples (i.e. $\boldsymbol{\varepsilon=0}$)
Number of rounds: $\mathbf{r = 2}$

[Ganguly'92, Afrati'10]

Example in one step



Find all triangles

$C_3(x,y,z) = R_1(x,y), R_2(y,z), R_3(z,x)$

R_1=

X	Y
a1	b3
a2	b1
a3	b2

R_2=

Y	Z
b1	c2
b2	c3
b3	c1

R_3=

Z	X
c1	a2
c2	a1
c3	a3

C_3 =

X	Y	Z
a3	b2	c3

Algorithm 2:

Servers form a cube: $[\mathbf{p}] \cong [\mathbf{p}^{1/3}] \times [\mathbf{p}^{1/3}] \times [\mathbf{p}^{1/3}]$

For each server $\mathbf{1} \leq \mathbf{u} \leq \mathbf{p}$:

Step 1: Choose random hash functions $\mathbf{h_1,h_2,h_3}$

send $\mathbf{R_1}(x,y)$ to

servers ($\mathbf{h_1}(x)$ mod $\mathbf{p}^{1/3}$, $\mathbf{h_2}(y)$ mod $\mathbf{p}^{1/3}$, *)

send $\mathbf{R_2}(y,z)$ to

servers (*, $\mathbf{h_2}(y)$ mod $\mathbf{p}^{1/3}$, $\mathbf{h_3}(z)$ mod $\mathbf{p}^{1/3}$)

send $\mathbf{R_3}(z,x)$ to

servers ($\mathbf{h_1}(x)$ mod $\mathbf{p}^{1/3}$, *, $\mathbf{h_3}(z)$ mod $\mathbf{p}^{1/3}$)

Output all triangles $\mathbf{R_1}(x,y)$, $\mathbf{R_2}(y,z)$, $\mathbf{R_3}(z,x)$

Load: $O(\mathbf{n/p} \times \mathbf{p}^{1/3})$ tuples (ε = **1/3**)

Number of rounds: **r = 1**

We Show

Find all triangles
$\mathbf{C_3(x,y,z) = R_1(x,y), R_2(y,z), R_3(z,x)}$

Load: $O(\mathbf{n/p} \times \mathbf{p^{1/3}})$ tuples ($\boldsymbol{\varepsilon} = \mathbf{1/3}$)
Number of rounds: $\mathbf{r} = \mathbf{1}$

Above algorithm is optimal for any randomized 1 round MPC algorithm for the triangle query

Based on general characterization of queries based on the *fractional cover number* of their associated hypergraph

Fractional Cover Number τ^*

Vertex Cover LP:

$\tau^* = \min \sum_i v_i$

Subject to:

$\sum_{x_i \in vars(R_j)} v_i \geq 1 \quad \forall j$

$v_i \geq 0 \quad \forall i$

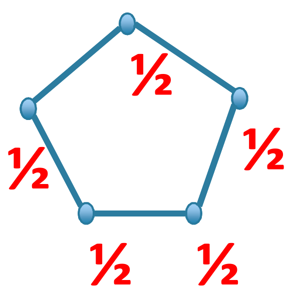
Edge Packing LP:

$\tau^* = \max \sum_j u_j$

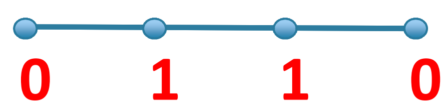
Subject to:

$\sum_{x_i \in vars(R_j)} u_j \leq 1 \quad \forall i$

$u_j \geq 0 \quad \forall j$



$\tau^*(C_k)=k/2$



$\tau^*(L_k)=\lceil k/2 \rceil$

1-Round No Skew

Theorem: Any **1-round** randomized **MPC** algorithm with $\mathbf{p=\omega(1)}$ and load $\mathbf{o(N/p^{1/\tau^*(Q)})}$ will fail to compute connected query **Q** on some matching database input with probability $\mathbf{\Omega(1)}$.

$\tau^*(C_3)=3/2$ so need $\Omega(N/p^{2/3})$ load, i.e. $\varepsilon \geq p^{1/3}$ for C_3

… previous 1-round algorithm is optimal

Can get a matching upper bound this for all databases without skew by setting parameters in randomized algorithm generalizing the triangle case

- exponentially small failure probability

HyperCube/Shares Algorithm

Vertex Cover LP:

$\tau^* = \min \sum_i v_i$

Subject to:

$$\sum_{x_i \in vars(R_j)} v_i \geq 1 \quad \forall j$$

$$v_i \geq 0 \quad \forall i$$

Algorithm: Decompose $p = (p^{v_1/\tau^*}, \dots, p^{v_m/\tau^*})$ and hash each tuple as in triangle case

1-Round No Skew

Theorem: Any **1-round** randomized **MPC** algorithm with $\mathbf{p=\omega(1)}$ and load $\mathbf{o(N/p^{1/\tau^*(Q)})}$ will fail to compute connected query **Q** on some matching database input with probability $\mathbf{\Omega(1)}$.

Follows from…

Lemma: For any *deterministic* **1-round MPC** algorithm, any processor that receives $\mathbf{O(N/p^{\delta})}$ bits about each input relation learns only $\mathbf{O(E[|Q(I)|]/p^{\tau^*(Q)\,\delta})}$ correct answers to connected query **Q** on average for a **random** matching database input **I**.

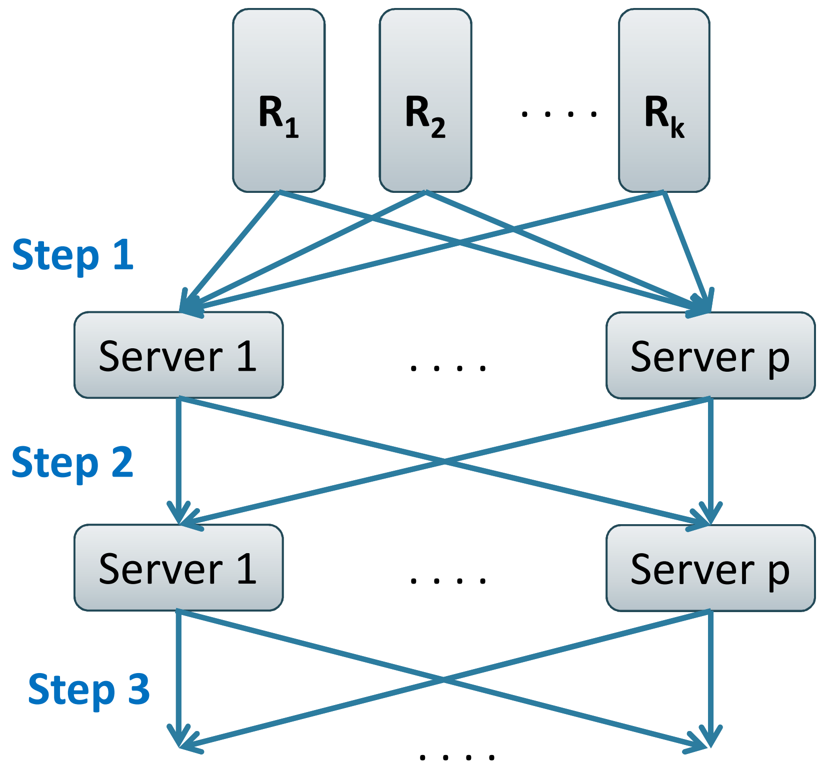
Communication Complexity Consequence

Whenever **$\tau^*(Q)>1$**, solving **Q** with **p** processors in one round requires **$\omega(N/p)$** bits received per processor where **N** = # input bits.

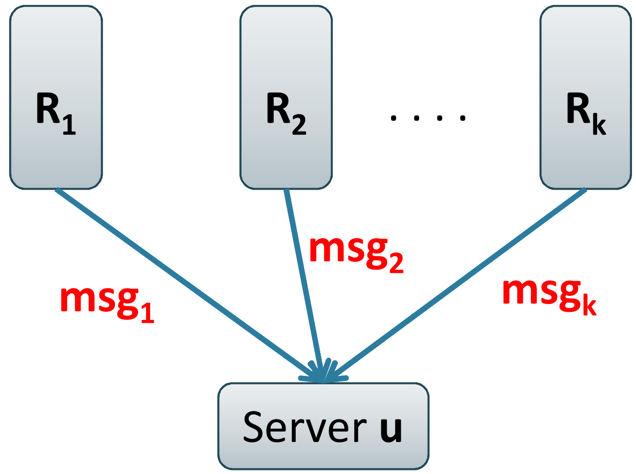
Lower bound implies failure even when total communication is **$\omega(N)$**

Slightly Stronger Lower Bound Model: MPC with Relation Servers

Input: each relation $\mathbf{R_1}$, $\mathbf{R_2}$, … , $\mathbf{R_k}$ is stored on a separate input server.
Step 1: each input server **i** distributes $\mathbf{R_i}$ to the **p** processors
Steps 2, 3, …: the **p** processors perform the computation as before



Information for a Single Processor



$I = (R_1, \ldots, R_k)$

$msg = (msg_1, \ldots, msg_k)$

$K_{msg_j}(R_j)$= set of tuples of R_j known by this processor

Prop: If $|msg|$ is $O(N/p^{\delta})$ then $E_I[\,|K_{msg(R_j(I))}(R_j)|\,]$ is $O(n/p^{\delta})$

$K_{msg}(Q)$= set of query answers known by this processor

$$= K_{msg_1}(R_1) \bowtie \ldots \bowtie K_{msg_k}(R_k)$$

Finishing off 1-Round Lower Bound

Lemma: In 1 round, for a processor that receives $\mathbf{O(N/p^{\delta})}$ bits about each input relation,

$$\mathbf{E_I[|K_{msg(I)}(Q)|]} \text{ is } \mathbf{O(E[|Q(I)|]/p^{\tau^*(Q)\,\delta})}.$$

Proof Ideas:

– Apply an inequality due to Friedgut to bound $\mathbf{E_I[|K_{\underline{msg}(I)}(Q)|]}$ in terms of $\mathbf{E_I[|\,K_{msg(R_j(I))}(R_j)|]}$

– Friedgut's inequality uses a *fractional edge cover* of **Q**

- Relate the fractional edge cover and the fractional edge packing number $\mathbf{\tau^*(Q)}$ to obtain the bound

Friedgut's Inequality

Given:

(Query) hypergraph $\boldsymbol{Q}$ with hyper-edges $\boldsymbol{S_j} \subseteq [k]$ for all $j \in [\ell]$

For all $\boldsymbol{a} \in [n]^k$ write $\boldsymbol{a_j}$ for the projection of $\boldsymbol{a}$ on coordinates of $\boldsymbol{S_j}$

Variables $w_j(\boldsymbol{a_j})$ for each $\boldsymbol{a_j} \in [n]^{\boldsymbol{S_j}}$

Then for any fractional *edge* cover $\boldsymbol{u} = (u_1, u_2, \ldots, u_\ell)$ of $\boldsymbol{Q}$

$$\sum_{\boldsymbol{a} \in [n]^k} \prod_{j=1}^{\ell} w_j(\boldsymbol{a_j}) \leq \prod_{j=1}^{\ell} \left(\sum_{\boldsymbol{a_j} \in [n]^k} w_j(\boldsymbol{a_j})^{\frac{1}{u_j}} \right)^{u_j}$$

Apply with $w_j(\boldsymbol{a_j})$ = Probability (over the input distribution) that processor learns that $\boldsymbol{a_j} \in \boldsymbol{R_j}$

LHS = Expected number of answer tuples processor learns

RHS = Product of independent quantities based on what a processor learns about each relation

Dealing with Skew (Irregular Hypergraphs)

- Suppose that in computing a join $\mathbf{R_1(x,y)R_2(y,z)}$ there is value **v** of attribute **y** that has very high degree in relation $\mathbf{R_1}$
- Then information about $\mathbf{R_1}$ tuples containing **v** must be spread out to multiple processors or there will be a hot spot of heavy load.
- However, without side information, the server for relation $\mathbf{R_2}$ will not know about this plan and will not know to replicate information about its tuples containing **v**

Dealing with Skew (Irregular Hypergraphs)

[BKoutrisSuciu 2014]

- Can quantify losses due to these hot spots and lack of coordination
- An alternative: Augment the 1-round MPC
  - Servers share identities and approximate degrees of "heavy hitter" vertices in any relation
    - those of degree $\geq \boldsymbol{m_j/p}$ where relation $\boldsymbol{j}$ has $\boldsymbol{m_j}$ tuples
    - $\boldsymbol{O(p)}$ such vertices in total
    - may be found by random sampling
  - Use this information to split up the processors into blocks and apply separate HyperCube algorithms on each block

Dealing with Skew (Irregular Hypergraphs)

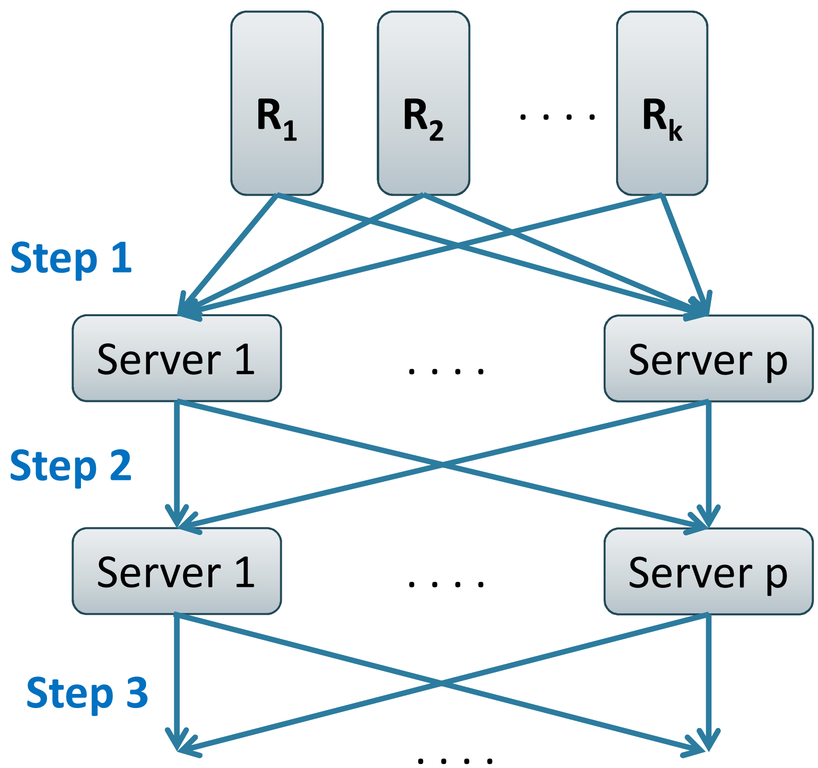
- With "heavy hitter" info can achieve, e.g.
  - For simple join with relation sizes $\boldsymbol{m_1}, \boldsymbol{m_2}$
    - Load $\mathbf{L} = \max\left(\frac{\boldsymbol{m_1}}{\boldsymbol{p}}, \frac{\boldsymbol{m_2}}{\boldsymbol{p}}, \sqrt{\frac{\boldsymbol{m_1 m_2}}{\boldsymbol{p}}}\right)$
  - For triangle join with relation sizes $\boldsymbol{m_1}, \boldsymbol{m_2}, \boldsymbol{m_3}$

    Load $\mathbf{L} =$

$$\max\left(\frac{\boldsymbol{m_1}}{\boldsymbol{p}}, \frac{\boldsymbol{m_2}}{\boldsymbol{p}}, \frac{\boldsymbol{m_3}}{\boldsymbol{p}}, \sqrt{\frac{\boldsymbol{m_1 m_2}}{\boldsymbol{p}}}, \sqrt{\frac{\boldsymbol{m_2 m_3}}{\boldsymbol{p}}}, \sqrt{\frac{\boldsymbol{m_1 m_3}}{\boldsymbol{p}}}, \frac{(\boldsymbol{m_1 m_2 m_3})^{\frac{1}{3}}}{\boldsymbol{p}^{\frac{2}{3}}}\right)$$

- Algorithms can be slowed down to work as sequential external memory algorithms and match known bounds
- Many cases remain open

Lower Bounds for Multiple Round MPC: A Circuit Complexity Barrier



In each round, each processor receives **L** bits of input and produces **L** bits of output so...

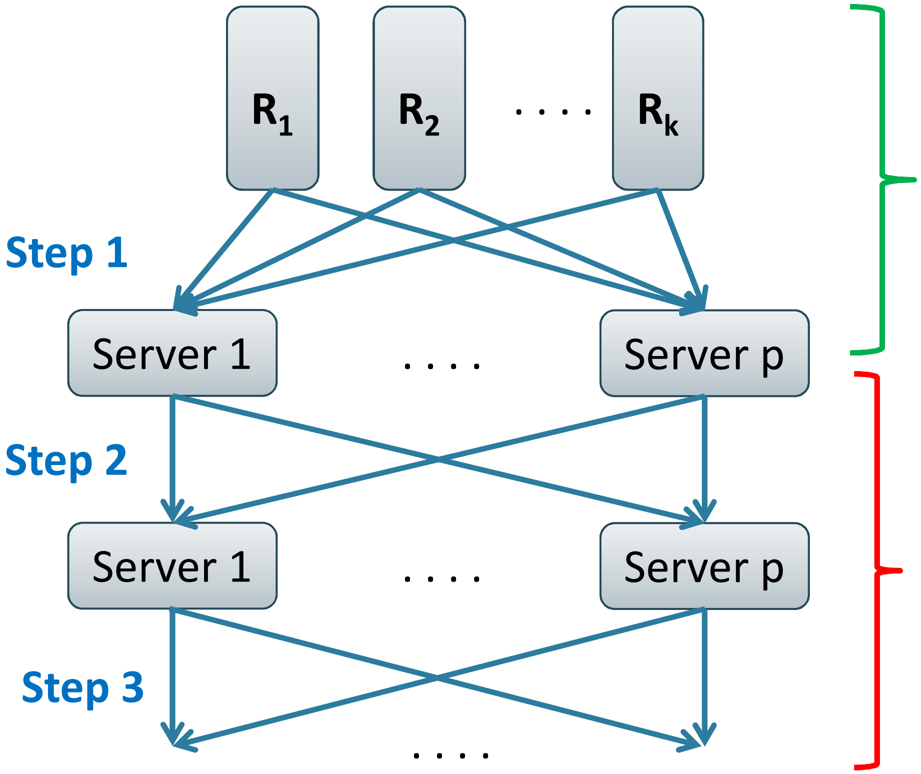
MPC in **r** rounds can simulate any circuit of depth **r** with **p** gates per level each of which computes an *arbitrary* function $g: \{0,1\}^L \rightarrow \{0,1\}^L$.

Just restricting the power of the processors doesn't do much to help avoid a similar circuit complexity barrier

Lower Bounds for Multiple Round MPC: Structured Model Required

- Problem: Messages are arbitrary bits that can give complex partial information
- Possible solution: restrict messages to database tuples
- Not enough: Set of tuples sent from A to B becomes their common knowledge after 1st round. Can be re-sent in later rounds to code arbitrary bits.
- Message routing also needs to be restricted

Lower Bounds for Multiple Rounds: Tuple-based MPC



Arbitrary bits sent

- Only (join) tuples① sent
- Routing based only on round #, tuple②, & step 1 messages from associated relations

① Join tuple: $[R_1(a,b)R_2(b,c)]$

② Or known join tuple containing it

e.g. can send $R_1(a,b)$ using $R_2(b,c)$

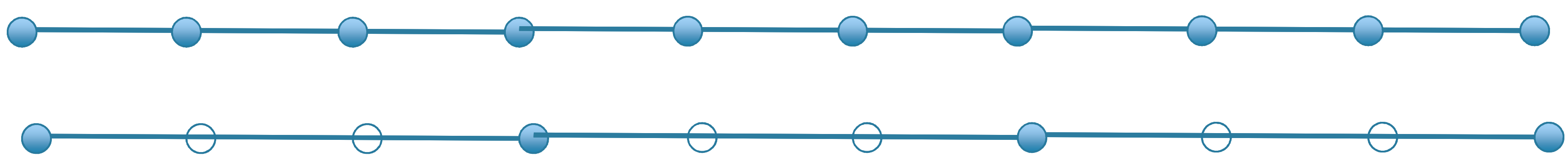
Algorithm for Multiple Rounds
No Skew

Let $k_\delta = \lfloor 2\delta \rfloor$.

Fact: k_δ = max {k : path query L_k doable in 1 round
with at most N/p^δ load}

Theorem: For connected Q there is a **simple** tuple-based MPC algorithm with $O(N/p^\delta)$ load on matching DBs using $r = \lceil \log \text{radius}(Q) / \log k_\delta \rceil + 1$ rounds

Idea: can effectively shrink paths by a factor k_δ per round.



radius(Q) = $\min_u \max_v d(u,v)$ where $d(u,v)$=# of edge hops

Lower Bound for Multiple Round Tuple-MPC Computing Tree-like Queries

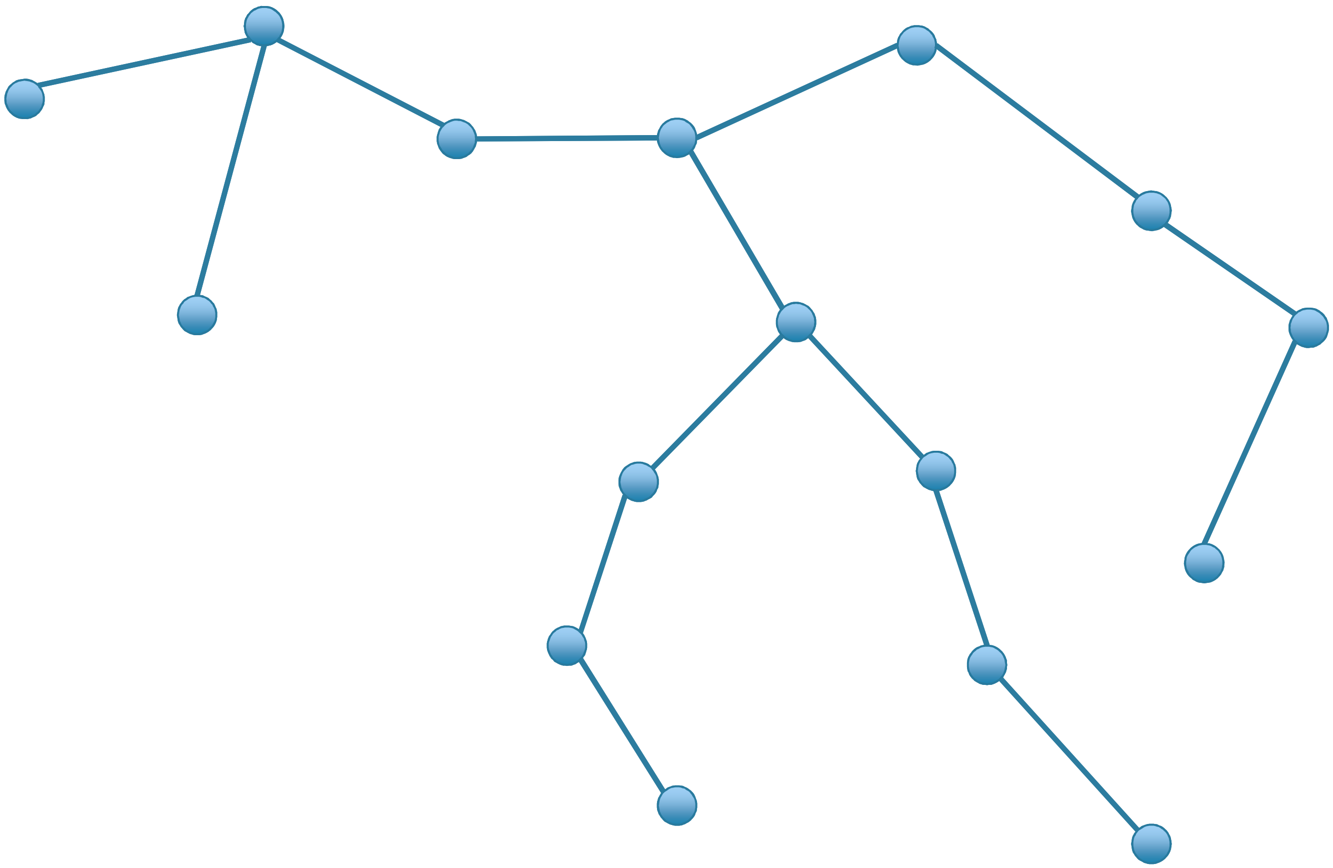
Theorem: For *tree-like* **Q** any tuple-based MPC algorithm with **$O(N/p^{\delta})$** load requires **$r \geq \log \text{diameter}(Q) / \log k_{\delta}$** rounds on matching databases.

Proof ideas:

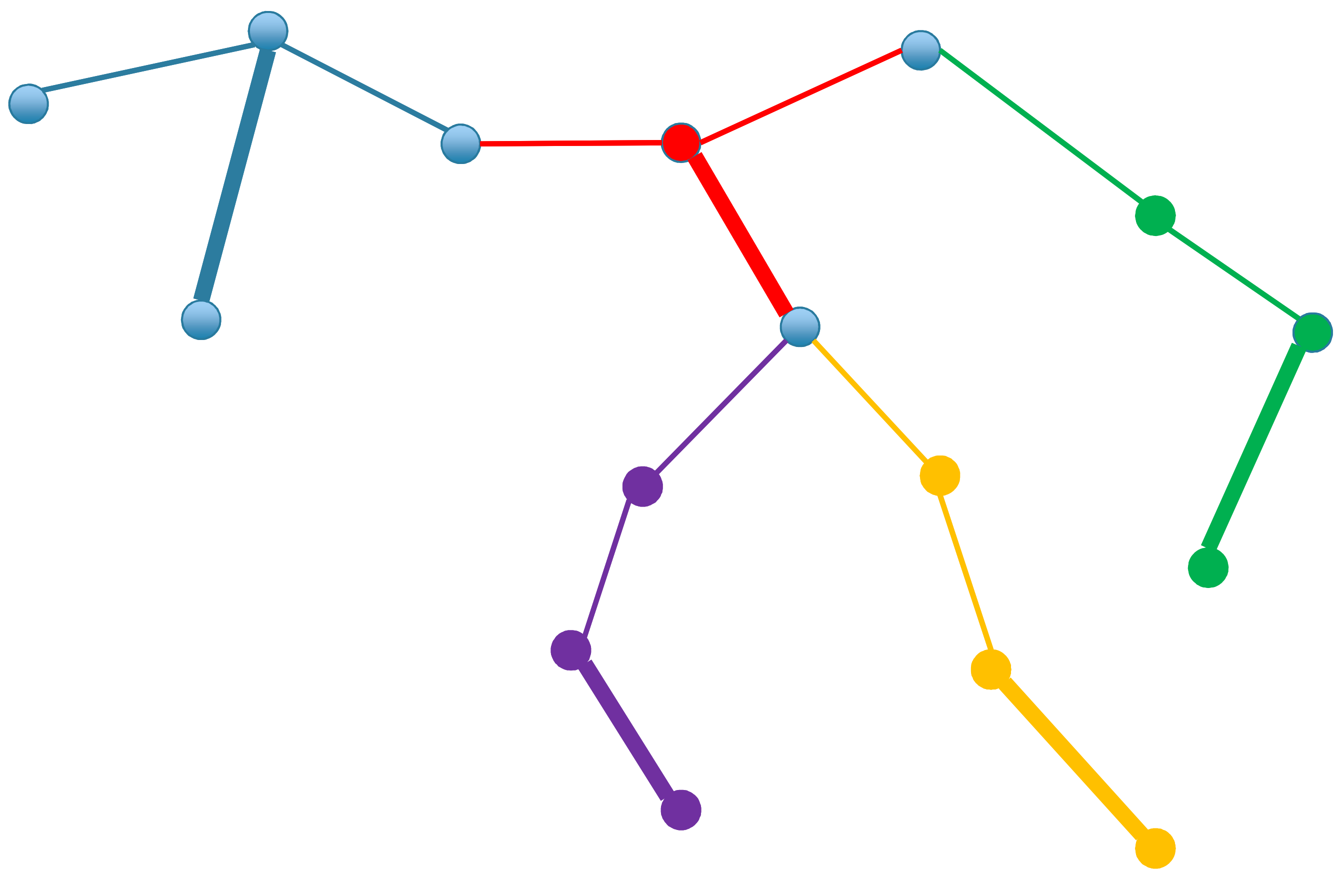
Inductively eliminate 2nd round until algorithm reduced to 1 round:

- In 2nd round can only send (join) tuples learned in 1st round
- By 1-round analysis, only a tiny number of join tuples learned in 1st round that are larger than **k_{δ}** - give away full extension of each to an answer for Q (reduces **n**)
- **Shrink** resulting graph so every join tuple of diameter **k_{δ}** has only one edge remaining. (Eliminates all joins from 1st round)
- All edges sent in 2nd round could have been sent in round 1

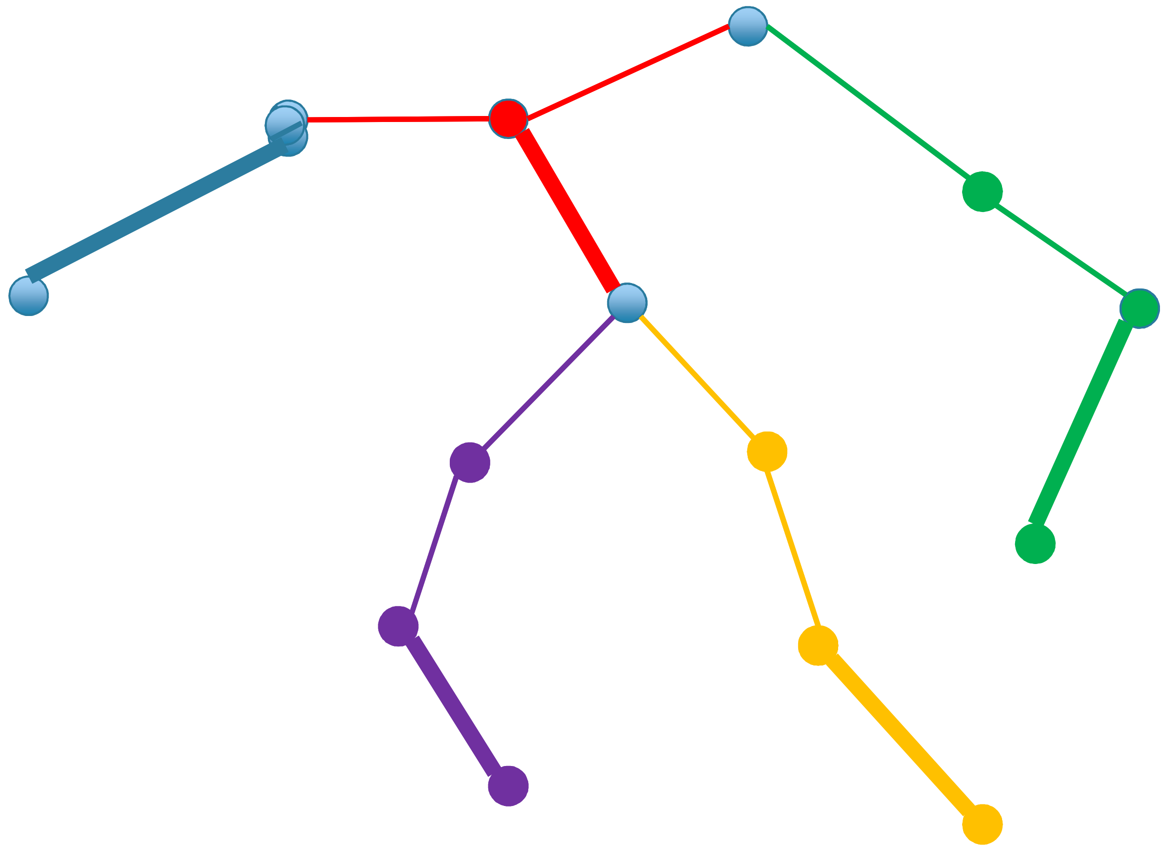
Shrinking the query graph



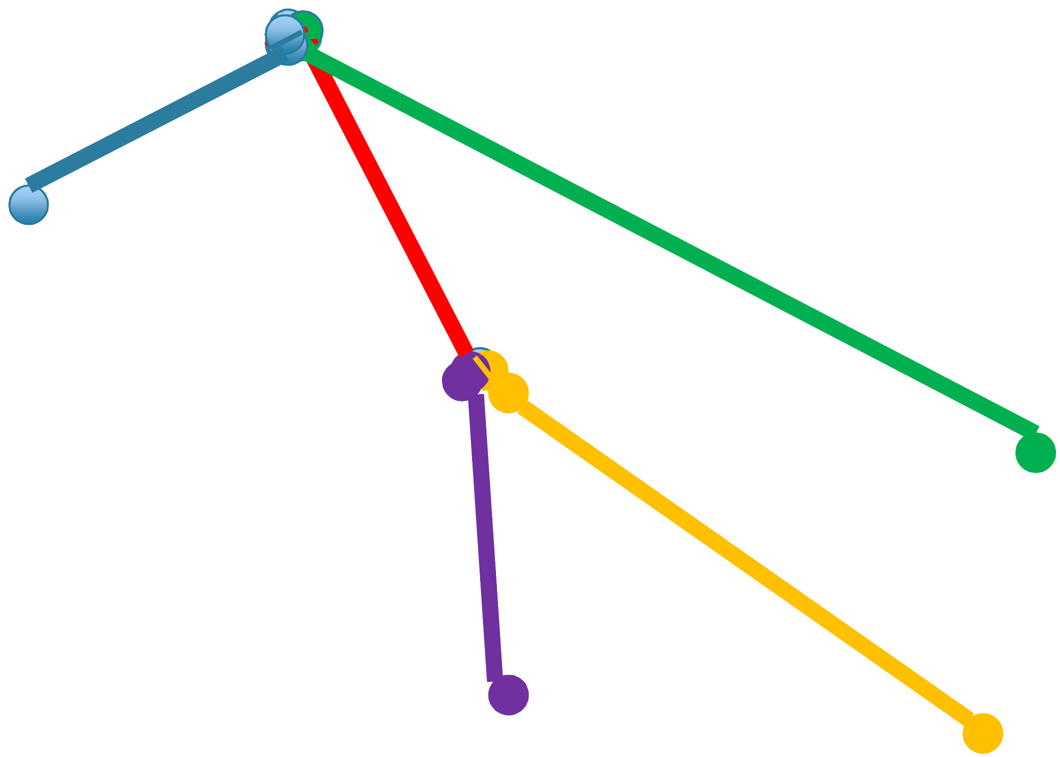
Shrinking the query graph



Shrinking the query graph



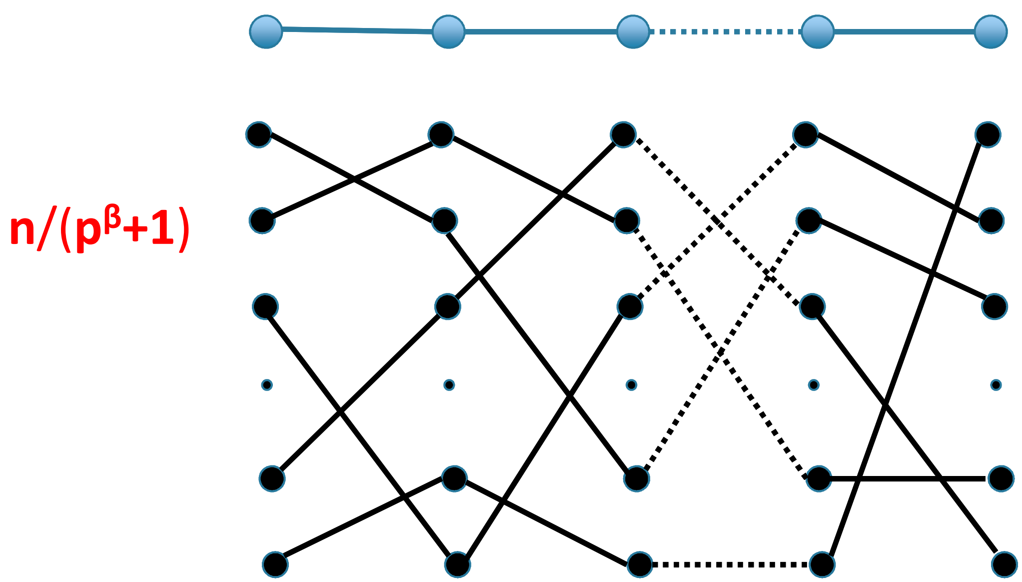
Shrinking the query graph



Corollary

Corollary: Any tuple-based MPC algorithm that finds st-paths in degree 2 undirected graphs with $O(N/p^{\delta})$ load for $\delta>0$ requires $\Omega(\log p)$ rounds.

Idea: reduction from line query L_k with $k=p^{\beta}$ for some small $\delta>0$



Open Problems

- Lower bounds for decision or counting problems?
  - All lower bounds known depend on the need to produce multiple outputs
- Handle skew efficiently in multi-round algorithms?
  - Intermediate results may be very large even when few answers
- Lower bounds in the full MPC model for 2 rounds?
  - Some depth 2 circuit lower bounds are known for arbitrary gates
- Multi-round algorithm lower bounds for non-tree examples like multi-round algorithms for examples like C_5
- Bounds for other kinds of problems in the MPC model

Thank you!

Fractional Cover Number τ^*

Vertex Cover LP:

$\tau^* = \min \sum_i v_i$

Subject to:

$\sum_{x_i \in vars(R_j)} v_i \geq 1 \quad \forall j$

$v_i \geq 0 \quad \forall i$

Edge Packing LP:

$\tau^* = \max \sum_j u_j$

Subject to:

$\sum_{x_i \in vars(R_j)} u_j \leq 1 \quad \forall i$

$u_j \geq 0 \quad \forall j$

Tight Edge Cover LP

Edge Packing LP:

$\tau^* = \max \sum_j u_j$

Subject to:

$\sum_{x_i \in vars(R_j)} u_j \leq 1 \quad \forall i$

$u_j \geq 0 \quad \forall j$

Tight Edge Cover:

Constraints:

$\sum_{x_i \in vars(R_j)} u_j + u_i^* = 1 \quad \forall i$

$u_j, u_i^* \geq 0 \quad \forall j \; \forall i$

Lower Bound: Apply Friedgut's Inequality using Tight Edge Cover solution. The slack variables u_i^* correspond to weights on new unary edges that don't affect probabilities.