Distributed Machine Learning

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Distributed Machine Learning

Modern applications: massive amounts of data distributed across multiple locations.

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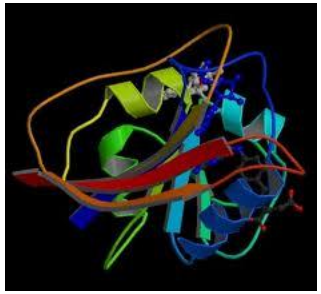
Modern applications: massive amounts of data distributed across multiple locations.

E.g.,

- video data

- scientific data



Key new resource communication.

This talk: models and algorithms for reasoning about communication complexity issues.

- Supervised Learning

  [Balcan-Blum-Fine-Mansour, COLT 2012] Runner UP Best Paper

- Clustering, Unsupervised Learning

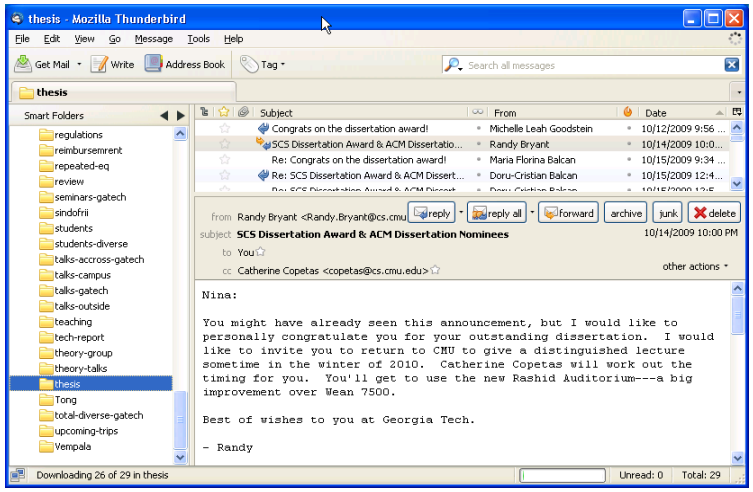
  [Balcan-Ehrlich-Liang, NIPS 2013]

  [Balcan-Kanchanapally-Liang-Woodruff, NIPS 2014]

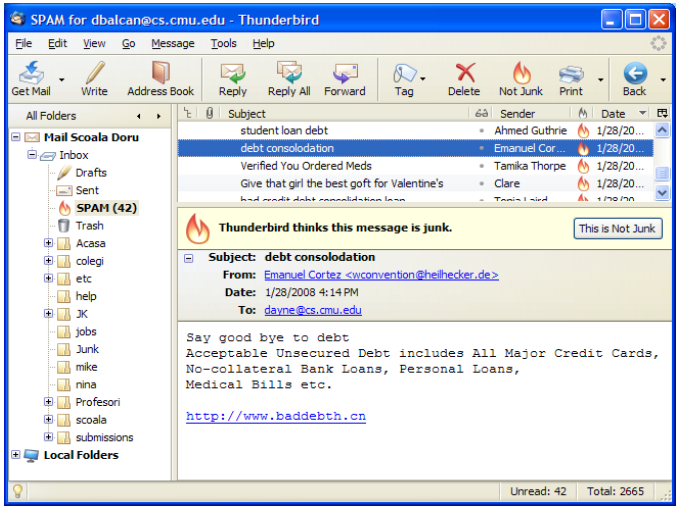
Supervised Learning

- E.g., which emails are spam and which are important.

Not spam



spam

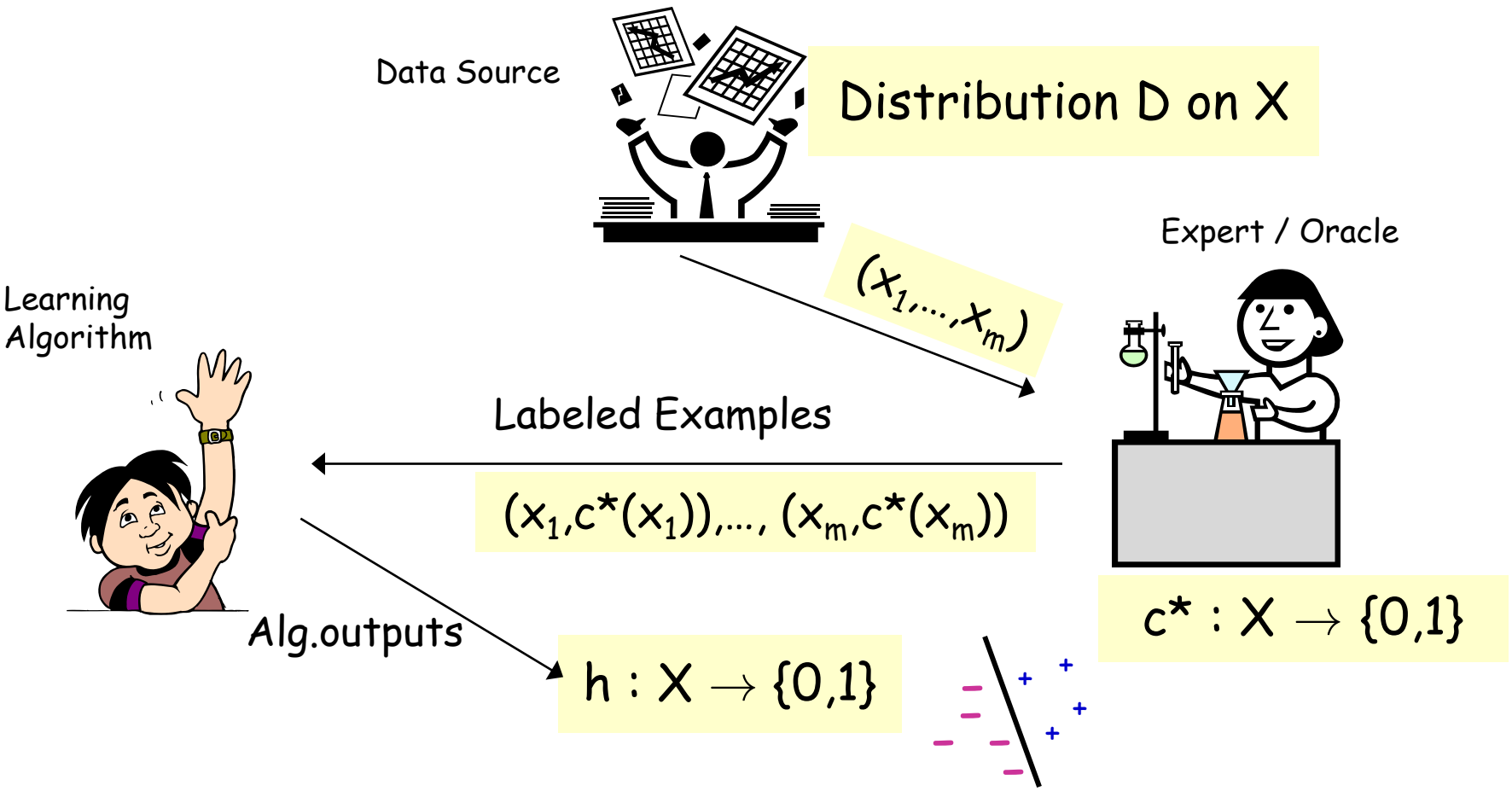


- E.g., classify objects as chairs vs non chairs.

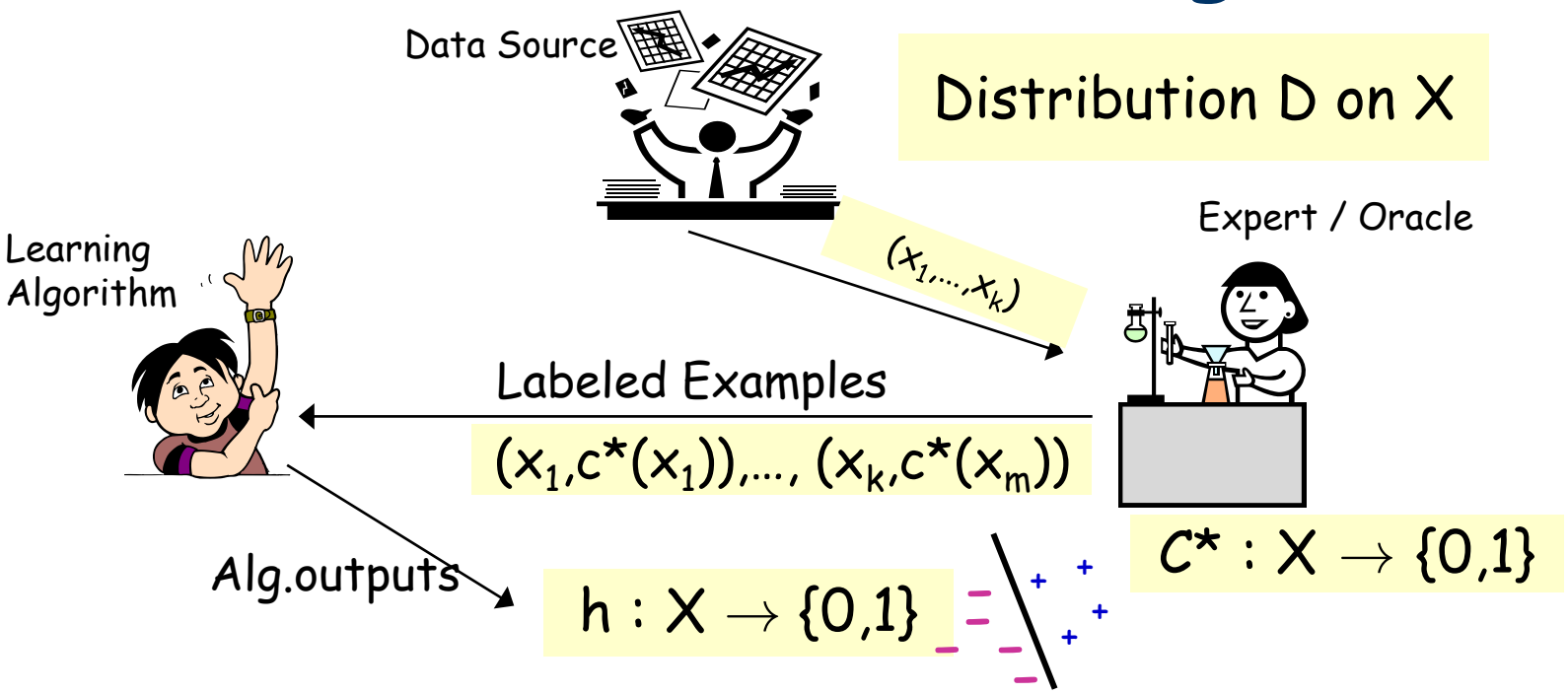
Not chair

chair

Statistical / PAC learning model



Statistical / PAC learning model



- Algo sees $(x_1,c^*(x_1)),\ldots,(x_k,c^*(x_m))$, x_i i.i.d. from D
- Do optimization over S, find hypothesis $h \in C$.
- Goal: h has small error over D.

$$err(h)=Pr_{x \in D}(h(x) \neq c^*(x))$$

- c^* in C, realizable case; else agnostic

Two Main Aspects in Classic Machine Learning

Algorithm Design. How to optimize?

Automatically generate rules that do well on observed data.

E.g., Boosting, SVM, etc.

Generalization Guarantees, Sample Complexity

Confidence for rule effectiveness on future data.

$$O\left(\frac{1}{\epsilon}\left(\mathrm{VCdim}(C)\log\left(\frac{1}{\epsilon}\right)+\log\left(\frac{1}{\delta}\right)\right)\right)$$

Distributed Learning

Many ML problems today involve massive amounts of data distributed across multiple locations.

Often would like low error hypothesis wrt the overall distrib.

Distributed Learning

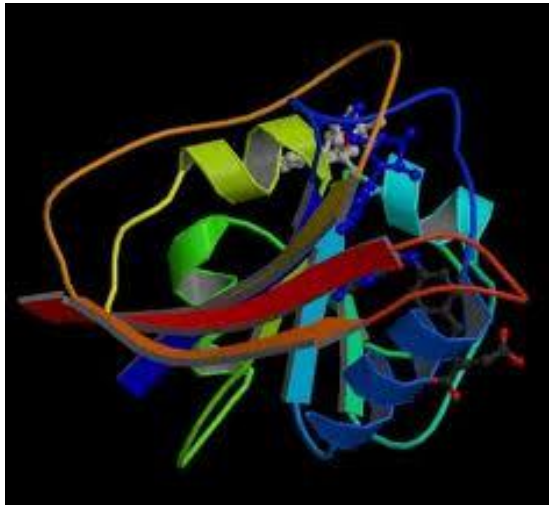
Data distributed across multiple locations.

E.g., medical data

Distributed Learning

Data distributed across multiple locations.

E.g., scientific data



Distributed Learning

- Data distributed across multiple locations.
- Each has a piece of the overall data pie.
- To learn over the combined D, must communicate.

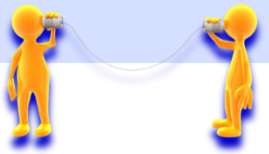
Important question: how much communication?

Plus, privacy & incentives.

Distributed PAC learning [Balcan-Blum-Fine-Mansour,COLT 2012]

- X – instance space. s players.
- Player i can sample from D_i, samples labeled by c*.
- Goal: find h that approximates c* w.r.t. D=1/s (D_1 + … + D_s)
- Fix C of VCdim d. Assume s << d. [realizable: c* ∈ C, agnostic:c*∉ C]

Goal: learn good h over D, as little communication as possible



- Total communication (bits, examples, hypotheses)
- Rounds of communication.

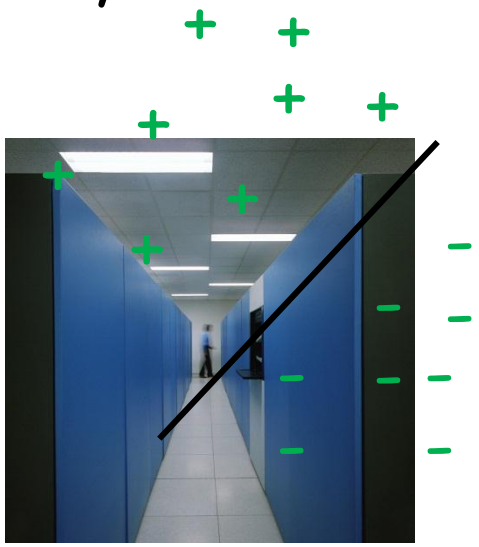
Efficient algos for problems when centralized algos exist.

Interesting special case to think about

s=2. One has the positives and one has the negatives.

- How much communication, e.g., for linear separators?

Player 1



Player 2

Overview of Our Results

Introduce and analyze Distributed PAC learning.

- Generic bounds on communication.
- Broadly applicable communication efficient distributed boosting.
- Tight results for interesting cases (conjunctions, parity fns, decision lists, linear separators over "nice" distrib).

Analysis of privacy guarantees achievable.

Some simple communication baselines.

Baseline #1

$d/\epsilon \log(1/\epsilon)$ examples, 1 round of communication

- Each player sends $d/(\epsilon s) \log(1/\epsilon)$ examples to player 1.
- Player 1 finds consistent $h \in C$, whp error $\leq \epsilon$ wrt D

...

Some simple communication baselines.

Baseline #2 (based on Mistake Bound algos):

M rounds, M examples & hyp, M is mistake-bound of C.

- In each round player 1 broadcasts its current hypothesis.
- If any player has a counterexample, it sends it to player 1. If not, done. Otherwise, repeat.

...

Some simple communication baselines.

Baseline #2 (based on Mistake Bound algos):

M rounds, M examples, M is mistake-bound of C.

- All players maintain same state of an algo A with MB M.
- If any player has an example on which A is incorrect, it announces it to the group.

...

Improving the Dependence on $1/\epsilon$

Baselines provide linear dependence in d and $1/\epsilon$, or M and no dependence on $1/\epsilon$.

Can get better $O(d \log 1/\epsilon)$ examples of communication!

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Recap of Adaboost

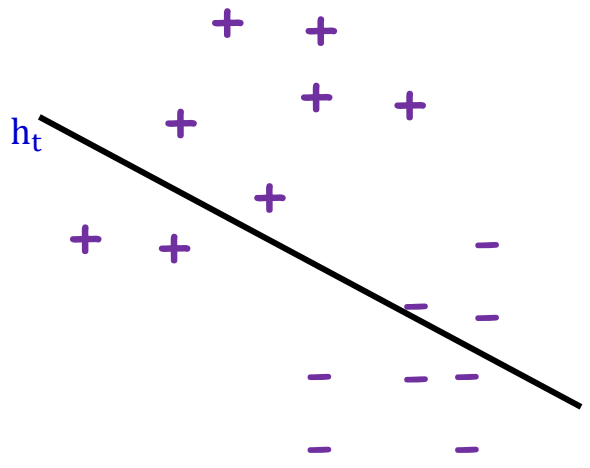
- Boosting: algorithmic technique for turning a weak learning algorithm into a strong (PAC) learning one.

Recap of Adaboost

- Boosting: turns a weak algo into a strong (PAC) learner.

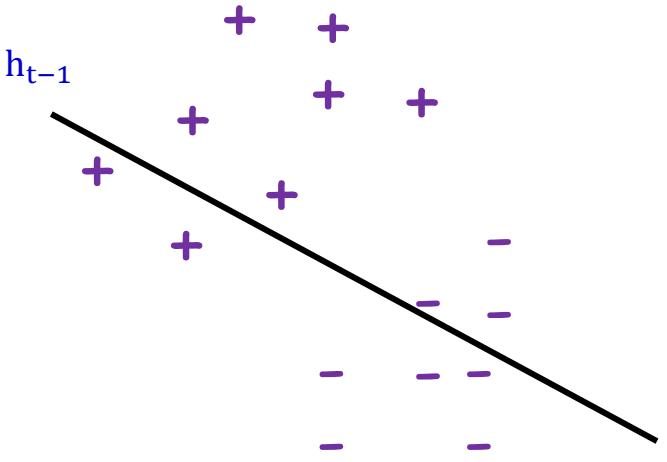
Input: $S=\{(x_1, y_1), \ldots, (x_m, y_m)\}$; weak learner A

- Weak learning algorithm A.
- For t=1,2, … ,T
  - Construct D_t on $\{x_1, \ldots, x_m\}$
  - Run A on D_t producing h_t
- Output H_final=sgn($\sum \alpha_t \ h_t$)



Recap of Adaboost

- Weak learning algorithm A.
- For t=1,2, … ,T
  - **Construct D_t on $\{x_1, \ldots, x_m\}$**
  - Run A on D_t producing h_t



- D_1 uniform on $\{x_1, \ldots, x_m\}$
- D_{t+1} increases weight on x_i if h_t incorrect on x_i ; decreases it on x_i if h_t correct.

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{-\alpha_t\}} \text{ if } y_i = h_t(x_i)$$

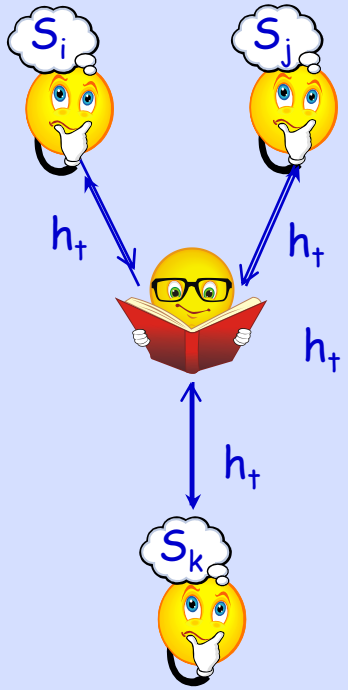
$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{\alpha_t\}} \text{ if } y_i \neq h_t(x_i)$$

Key points:

- $D_{t+1}(x_i)$ depends on $h_1(x_i), \ldots, h_t(x_i)$ and normalization factor that can be communicated efficiently.
- To achieve weak learning it suffices to use O(d) examples.

Distributed Adaboost

- Each player i has a sample S_i from D_i.
- For $t=1,2, \ldots, T$
  - Each player sends player 1, enough data to produce weak hyp h_t.
    [For t=1, O(d/s) examples each.]
  - Player 1 broadcasts h_t to other players.



Distributed Adaboost

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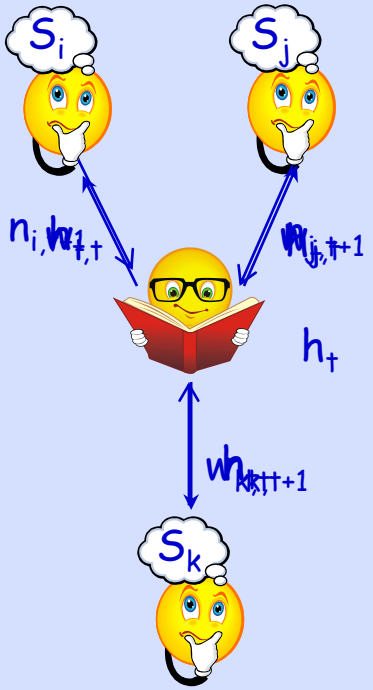
- For $t=1,2,\ldots,T$
  - Each player sends player 1, enough data to produce weak hyp h_t.
    [For t=1, O(d/s) examples each.]
  - Player 1 broadcasts h_t to other players.
  - Each player i reweights its own distribution on S_i using h_t and sends the sum of its weights $w_{i,t}$ to player 1.
  - Player 1 determines the #of samples to request from each i [samples $O(d)$ times from the multinomial given by $w_{i,t}/W_t$].



Distributed Adaboost

Can learn any class C with $O(\log(1/\epsilon))$ rounds using $O(d)$ examples + $O(s \log d)$ bits per round.

[efficient if can efficiently weak-learn from $O(d)$ examples]

Proof:

- As in Adaboost, $O(\log 1/\epsilon)$ rounds to achieve error ϵ.
- Per round: $O(d)$ examples, $O(s \log d)$ extra bits for weights, 1 hypothesis.

Dependence on $1/\epsilon$, Agnostic learning

Distributed implementation of Robust halving [Balcan-Hanneke'12].

- error $O(OPT)+\epsilon$ using only $O(s \log|C| \log(1/\epsilon))$ examples.

Not computationally efficient in general, but says $O(\log(1/\epsilon))$ possible in principle.

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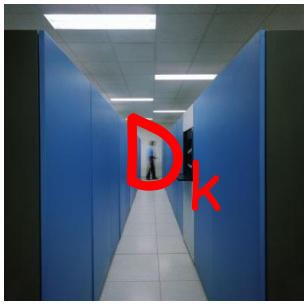
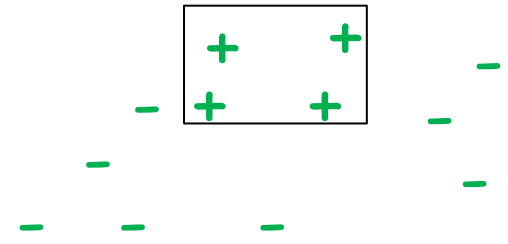


Better results for special cases

Intersection-closed when fns can be described compactly .



C is intersection-closed, then C can be learned in one round and s hypotheses of total communication.

Algorithm:

- Each i draws S_i of size $O(d/\epsilon \log(1/\epsilon))$, finds smallest h_i in C consistent with S_i and sends h_i to player 1.
- Player 1 computes smallest h s.t. $h_i \subseteq h$ for all i.

Key point:

h_i, h never make mistakes on negatives, and on positives h could only be better than h_i ($\mathrm{err}_{D_i}(h) \leq \mathrm{err}_{D_i}(h_i) \leq \epsilon$)

Better results for special cases

E.g., conjunctions over $\{0,1\}^d$ [$f(x) = x_2x_5x_9x_{15}$]

- Only $O(s)$ examples sent, $O(sd)$ bits.
  - Each entity intersects its positives.
  - Sends to player 1.
  - Player 1 intersects & broadcasts.

1101111011010111
1111110111001110
1100110011001111
1100110011000110

[Generic methods $O(d)$ examples, or $O(d^2)$ bits total.]

Interesting class: parity functions

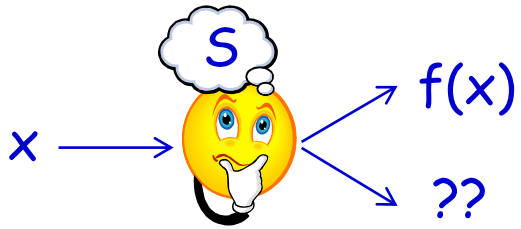
- $s = 2, X = \{0,1\}^d$, C = parity fns, $f(x) = x_{i_1} \text{XOR } x_{i_2} \ldots \text{XOR } x_{i_l}$
- Generic methods: O(d) examples, $O(d^2)$ bits.
- Classic CC lower bound: $\Omega(d^2)$ bits LB for proper learning.

Improperly learn C with O(d) bits of communication!

Key points:

- Can properly PAC-learn C.
  [Given dataset S of size $O(d/\epsilon)$, just solve the linear system]

- Can non-properly learn C in reliable-useful manner [RS'88]



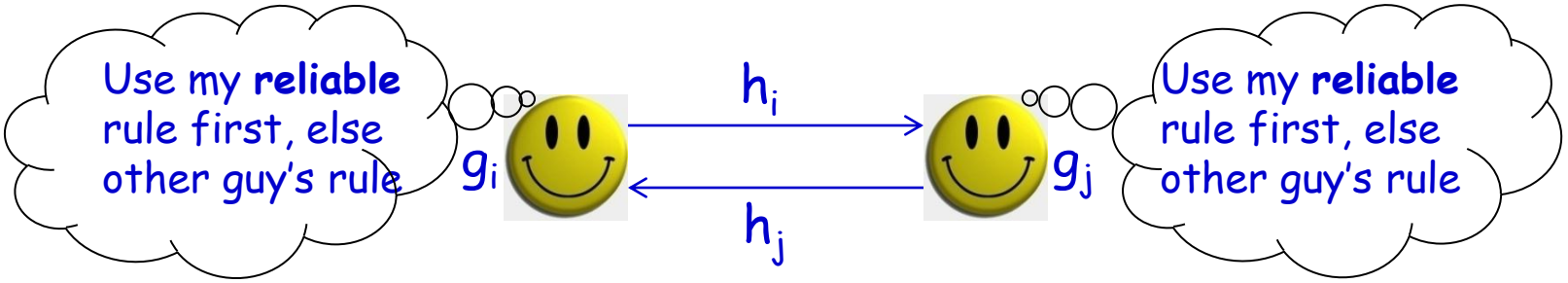
[if x in subspace spanned by S, predict accordingly, else say "?"]

Interesting class: parity functions

Improperly learn C with $O(d)$ bits of communication!

Algorithm:

- Player i properly PAC-learns over D_i to get parity h_i. Also improperly R-U learns to get rule g_i. Sends h_i to player j.
- Player i uses rule R_i: "if g_i predicts, use it; else use h_j"



Key point: low error under D_j because h_j has low error under D_j and since g_i never makes a mistake putting it in front does not hurt.

Distributed PAC learning: Summary

- First time consider communication as a fundamental resource.

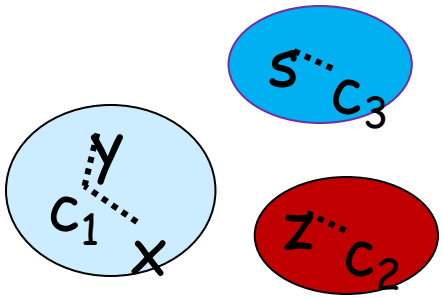
- General bounds on communication, communication-efficient distributed boosting.

- Improved bounds for special classes (intersection-closed, parity fns, and linear separators over nice distributions).

Distributed Clustering

[Balcan-Ehrlich-Liang, NIPS 2013]

[Balcan-Kanchanapally-Liang-Woodruff, NIPS 2014]

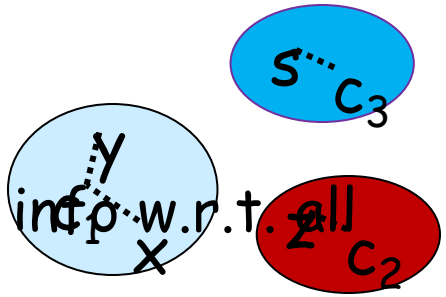


Center Based Clustering

k-median: find center pts $c_1, c_2, \ldots, c_k$ to minimize $\sum_x \min_i d(x,c_i)$

k-means: find center pts $c_1, c_2, \ldots, c_k$ to minimize $\sum_x \min_i d^2(x,c_i)$

Key idea: use coresets.



Coresets short summaries capturing relevant info w.r.t. all clusterings.

Def: An ϵ-coreset for a set of pts S is a set of points $\tilde{S}$ s.t. and weights $w: \tilde{S} \to R$ s.t. for any sets of centers $\mathbf{c}$:

$$(1-\epsilon)\text{cost}(S,\mathbf{c}) \leq \sum_{p\in D} w_p \text{cost}(p,\mathbf{c}) \leq (1+\epsilon)\text{cost}(S,\mathbf{c})$$

Algorithm (centralized)

- Find a coreset $\tilde{S}$ of S. Run an approx. algorithm on $\tilde{S}$.

Distributed Clustering [Balcan-Ehrlich-Liang, NIPS 2013]

k-median: find center pts $c_1, c_2, \ldots, c_k$ to minimize $\sum_x \min_i d(x,c_i)$

k-means: find center pts $c_1, c_2, \ldots, c_k$ to minimize $\sum_x \min_i d^2(x,c_i)$

- Key idea: use coresets, short summaries capturing relevant info w.r.t. all clusterings.
- [Feldman-Langberg STOC'11] show that in centralized setting one can construct a coreset of size $O(kd/\epsilon^2)$
- By combining local coresets, get a global coreset; the size goes up multiplicatively by s.
- In [Balcan-Ehrlich-Liang, NIPS 2013] show a two round procedure with communication only $O(kd/\epsilon^2 + sk)$

  [As opposed to $O(s\,kd/\epsilon^2)$]

Clustering, Coresets [Feldman-Langberg'11]

[FL'11] construct in centralized cases a coreset of size $O(kd/\epsilon^2)$.

1. Find a constant factor approx. B, add its centers to coreset
   [this is already a very coarse coreset]
2. Sample $O(kd/\epsilon^2)$ pts according to their contribution to the cost of that approximate clustering B.

Key idea: one way to think about this construction

- Upper bound penalty we pay for p under any set of centers **c** by distance between p and its closest center b_p in B
  - For any set of centers $\mathbf{c}$, penalty we pay for point p under any set of centers c, $f(p) = cost(p, \mathbf{c}) - cost(b_p, \mathbf{c})$
  - Note $f(p) \in [-cost(p, b_p), cost(p, b_p)]$.
    This motivates sampling according to $cost(p, b_p)$

Distributed Clustering [Balcan-Ehrlich-Liang, NIPS 2013]

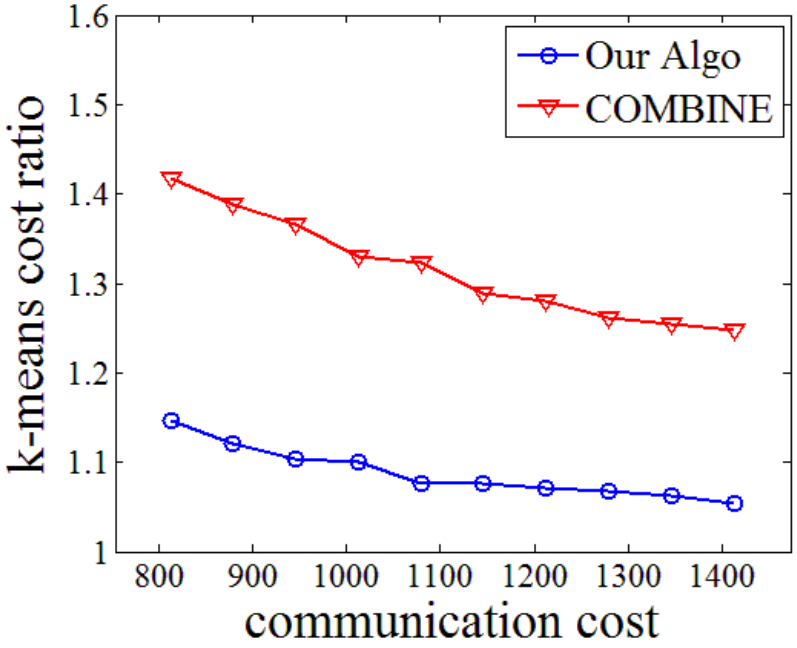
Feldman-Langberg'11 show that in centralized setting one can construct a coreset of size $O(kd/\epsilon^2)$.

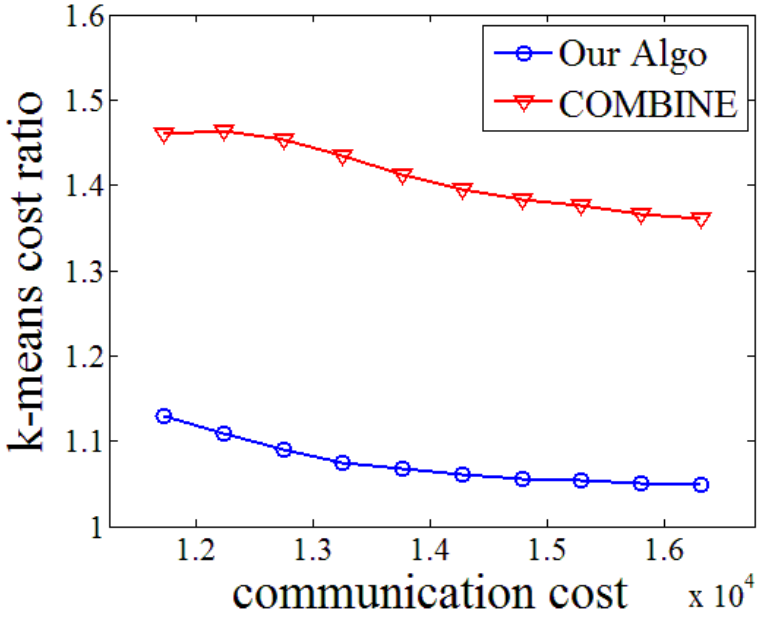
Key idea: in distributed case, show how to do this using only local constant factor approx.

1. Each player, finds a local constant factor approx. B_i and sends cost(B_i, P_i) and the centers to the center.
2. Center sample $n = O(kd/\epsilon^2)$ pts $n = n_1 + \cdots + n_s$ from multinomial given by these costs.
3. Each player i sends n_i points from P_i sampled according to their contribution to the local approx.

Distributed Clustering [Balcan-Ehrlich-Liang, NIPS 2013]

k-means: find center pts $c_1, c_2, \ldots, c_k$ to minimize $\sum_x \min_i d^2(x,c_i)$





Open questions (Learning and Clustering)

- Efficient algorithms in noisy settings; handle failures, delays.
- Even better dependence on $1/\epsilon$ for communication efficiency for clustering via boosting style ideas.
    - Can use distributed dimensionality reduction to reduce dependence on d. [Balcan-Kanchanapally-Liang-Woodruff, NIPS 2014]
- More refined trade-offs between communication complexity, computational complexity, and sample complexity.