Tight Bounds for Linear Sketches of Approximate Matchings

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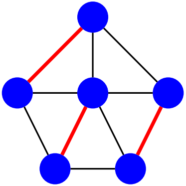
Joint work with Sanjeev Khanna (Penn),Yang Li (Penn), and Grigory Yaroslavtsev (Penn)

Matchings in Graphs

- Matching: A collection of vertex-disjoint edges.



- Perfect Matching: Every vertex is in the matching.

Maximum Matching problem: Find a matching with a largest number of edges.

Matchings in Graphs

Maximum matching is a fundamental problem with many applications.

- Many celebrated algorithms for matchings: Ford-Fulkerson, Edmond's, Hopcroft-Karp, Mucha-Sankowski, Madry's, ...
- Studied in various computational models: distributed, dynamic, online, streaming, ...

This talk: sublinear space algorithms for computing approximate matchings in dynamic graph streams.

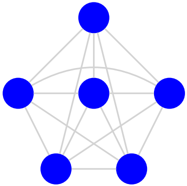
Dynamic Graph Streams

- The input graph is presented as a sequence of edge insertions and deletions.

Stream:

Edge-frequency vector:

$\vec{f} = \left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right]$



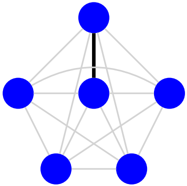
Dynamic Graph Streams

- The input graph is presented as a sequence of edge insertions and deletions.

Stream: $+e_1$

Edge-frequency vector:

$$\vec{f} = \left[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right]$$



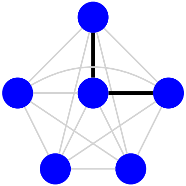
Dynamic Graph Streams

- The input graph is presented as a sequence of edge insertions and deletions.

Stream: $+e_1, +e_7$

Edge-frequency vector:

$$\vec{f} = \left[1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0\right]$$



Dynamic Graph Streams

- The input graph is presented as a sequence of edge insertions and deletions.

Stream: $+e_1, +e_7, +e_{11}$

Edge-frequency vector:

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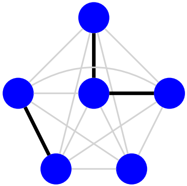

Dynamic Graph Streams

- The input graph is presented as a sequence of edge insertions and deletions.

Stream: $+e_1, +e_7, +e_{11}, -e_1$

Edge-frequency vector:

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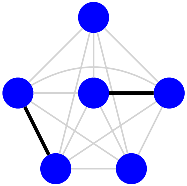

Dynamic Graph Streams

- The input graph is presented as a sequence of edge insertions and deletions.
- Algorithm makes a single pass over the entire input but only has a small space to store information about the input as it passes by.
- At the end of the sequence, the algorithm outputs a solution using the stored information.

Linear Sketches

For a graph G with n vertices:

- Let $\vec{f}$ be the edge-frequency vector representing G.
- Let M be an $s \times n^2$ dimensional matrix (possibly randomly chosen) for some parameter s.
- The s-dimensional vector $M \cdot \vec{f}$ is a linear sketch of G:

$$\Big[\quad M \quad \Big]_{s \times n^2} \cdot \Big[\vec{f} \Big]_{n^2} = \Big[\; \Big]_{s}$$

- Requires $O(s)$ for storage $\implies$ $O(s)$ size for storing the graph instead of $O(n^2)$.

Linear Sketches and Dynamic Graph Streams

Linear sketches are main technique for computing in dynamic graph streams:

- Maintain a linear sketch of the input graph during the stream.
  - When an edge e_i is updated: $M \cdot (\overrightarrow{f} \pm \overrightarrow{e_i}) = M \cdot \overrightarrow{f} \pm M \cdot \overrightarrow{e_i}$
- At the end of the stream, apply an arbitrary function to $M \cdot \overrightarrow{f}$ to compute the answer.
- Space requirement of the algorithm:
  $O(s)$ for the linear sketch + random bits needed for storing M implicitly.

Dynamic graph stream algorithms and linear sketches are (essentially) equivalent [AHLW16, LNW14].

Results in Dynamic Graph Streams

Linear sketches proved to be useful for various graph problems:

- Connectivity, edge connectivity, minimum spanning tree, spectral sparsification, triangle counting, densest subgraph, . . .
- Most of them have essentially the same space requirement as the best streaming algorithm for insertion-only streams.

An important missing problem is the maximum matching problem.

Matching in Graph Streams

Insertion-only streams:

- Exact computation requires $\Omega(n^2)$ space [FKM+05].
- 2-approximation in $O(n)$ space is trivial but no better than 2-approximation in $o(n^2)$ space is known.
- Beating $\frac{e}{e-1}$-approximation requires $n^{1+\Omega(1/\log\log n)}$ space [Kap13, GKK12].
- Lots and lots of other results: [McG05] [FKM+05] [EKS09] [ELMS11] [GKK12] [KMM12] [Zel12] [AGM12] [AG13b] [Kap13] [GO13] [KKS14] [CS14] [EHL+15] [AG13a] ...

Dynamic graph streams:

- Prior to our work, no non-trivial results were known for single-pass algorithms.

Our Results

We provide a complete resolution of matchings in dynamic graph streams:

Theorem (Upper bound)

For any $0 \leq \epsilon \leq 1/2$, space of $\tilde{O}(n^{2-3\epsilon})$ is sufficient for computing an n^{ϵ}-approximate maximum matching.

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Theorem (Lower bound)

For any $\epsilon \geq 0$, space of $\tilde{\Omega}(n^{2-3\epsilon})$ is necessary for computing an n^{ϵ}-approximate maximum matching.

For $\epsilon > 1/2$, $\tilde{O}(n^{1-\epsilon})$ space is necessary and sufficient for an n^{ϵ}-approximation.

Recent Related Work

Two recent results obtained independently and concurrently:

	Upper bound	Lower bound
[Kon15]	$\tilde{O}(n^{2-2\epsilon})$	$\tilde{\Omega}(n^{3/2-4\epsilon})$
[CCE+16]	$\tilde{O}(n^{2-3\epsilon})$ $(\epsilon \leq 1/2)$	-
This work	$\tilde{O}(n^{2-3\epsilon})$ $(\epsilon \leq 1/2)$ $\tilde{O}(n^{1-\epsilon})$ $(\epsilon > 1/2)$	$\tilde{\Omega}(n^{2-3\epsilon})$

Upper Bound

n^ϵ-Approximation for Matchings

Theorem (Upper bound)

For any $0 < \epsilon \leq 1/2$, space of $\tilde{O}(n^{2-3\epsilon})$ is sufficient for computing an n^ϵ-approximate maximum matching in dynamic graph streams.

- The algorithm needs only to store a linear sketch.
- W.l.o.g. we can restrict our attention to bipartite graphs.
- For simplicity, assume there is a perfect matching $M^\star$ in the input graph $G(L, R, E)$.

ℓ_0-sampler

Problem: How can we recover one edge from the edge-frequency vector $\vec{f}$ of G defined by a dynamic graph stream in a small space?

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Solution:

1. Let $M = \left[1, 2, \ldots, n^2\right]$.
2. Return $M \cdot \overrightarrow{f}$.

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Toy problem 2: If there is exactly one edge in G return it, otherwise output FAIL.

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Solution:

1. Let $M = \begin{bmatrix} 1, 2, \ldots, n^2 \\ 1, 1, \ldots, 1 \end{bmatrix}$.
2. Let $\begin{bmatrix} x \\ y \end{bmatrix} = M \cdot \overrightarrow{f}$; if $y = 1$ return x, otherwise output FAIL.

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Toy problem 3: Suppose there are exactly D edges in G, i.e., $\|\overrightarrow{f}\|_0 = D$; return one edge in G w.p. $2/3$, otherwise output FAIL.

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Solution:

1. Randomly sample $\frac{D}{n^2}$ edge slots from G, i.e., $\frac{1}{D}$ fraction of rows in $\overrightarrow{f}$.
2. Run the algorithm from the previous part over the sub-sampled graph.

ℓ_0-sampler

Problem: How can we recover one edge from the edge-frequency vector $\overrightarrow{f}$ of G defined by a dynamic graph stream in a small space?

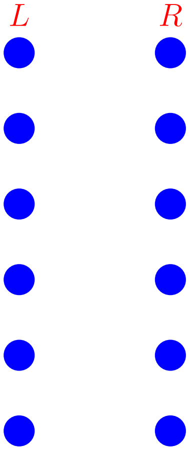
Theorem ([JST11])

There exists a distribution of over $\text{polylog}(N) \times N$ dimensional matrices M, such that for any $x \in \mathbb{R}^N$, one random non-zero element of x can be reconstructed from $M \cdot x$ w.h.p.

ℓ_0-samplers allow us to recover an edge between any two groups of pre-specified vertices in dynamic graph streams.

Warm-up: An $\tilde{O}(n^{2-2\epsilon})$ Space Algorithm

1. Group vertices in L and R into $n^{1-\epsilon}$ groups.



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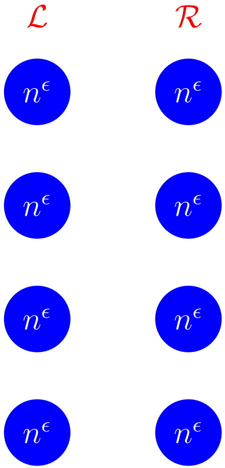

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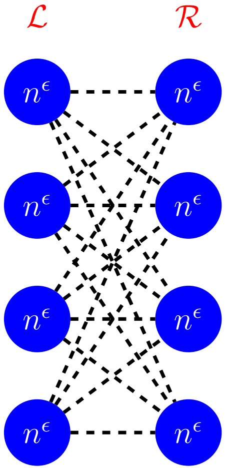

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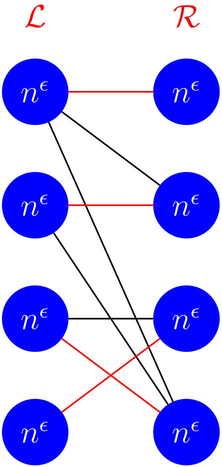


Analysis of the $\tilde{O}(n^{2-2\epsilon})$ Space Algorithm

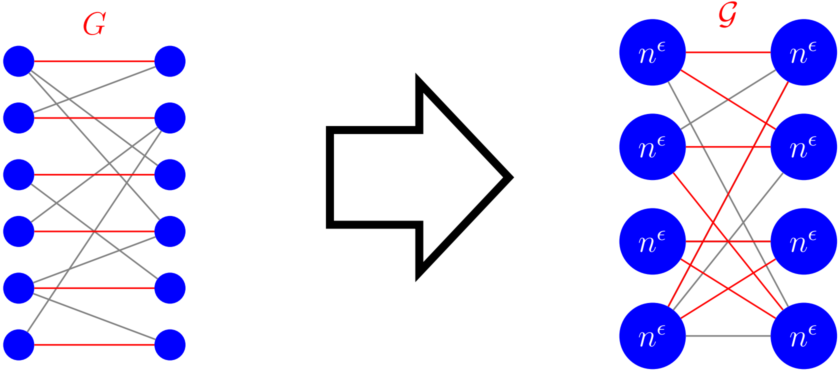
Space requirement:

- We picked $n^{2-2\epsilon}$ ℓ_0-samplers: one per each pair in $(\mathcal{L}, \mathcal{R})$.
- Each ℓ_0-sampler requires $\mathrm{polylog}(n)$ space.
- Total of $\tilde{O}(n^{2-2\epsilon})$ space.

Analysis of the $\tilde{O}(n^{2-2\epsilon})$ Space Algorithm

Approximation factor:

- The perfect matching $M^\star$ in G induces an n^ϵ-regular (multi-)graph in the grouped graph $\mathcal{G}$.



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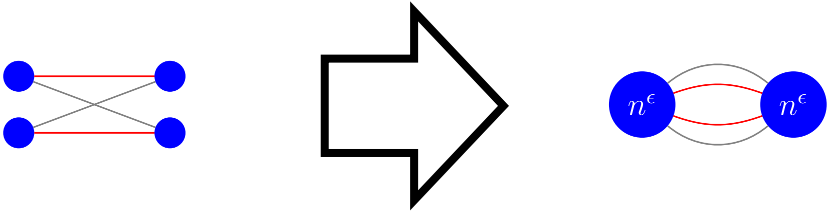

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- The sampled edges have a matching of size $n^{1-\epsilon}$, i.e., an n^ϵ-approximate maximum matching.

Improving to an $\tilde{O}(n^{2-3\epsilon})$ Space Algorithm

Insight:

- The graph $\mathcal{G}$ in the previous algorithm is an n^{ϵ}-regular (multi-)graph.
- Any n^{ϵ}-regular graph has n^{ϵ} edge-disjoint perfect matching.
- The previous algorithm focused only on a single perfect matching in $\mathcal{G}$.

Can we sub-sample edges of $\mathcal{G}$ while still maintaining a large matching in the sampled graph?

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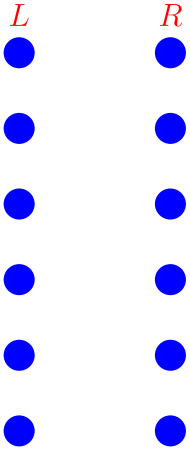

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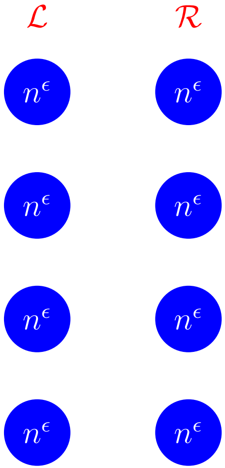

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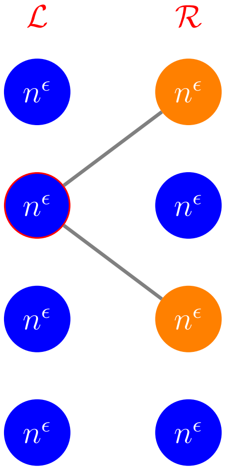

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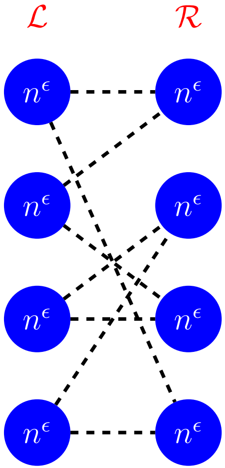

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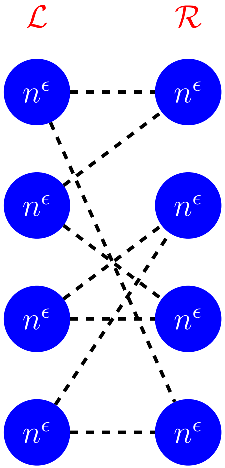

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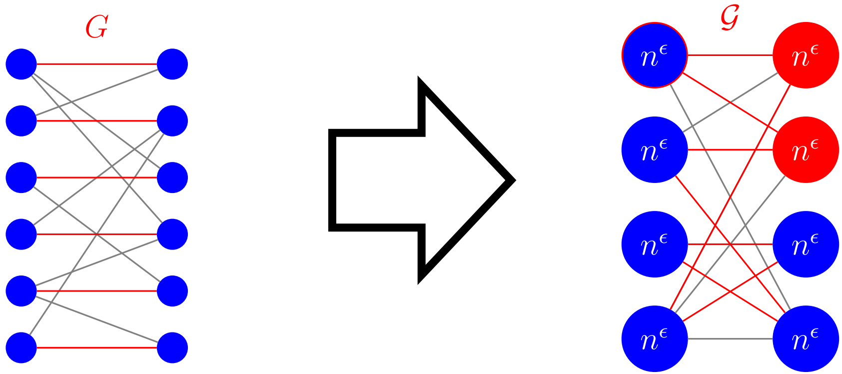

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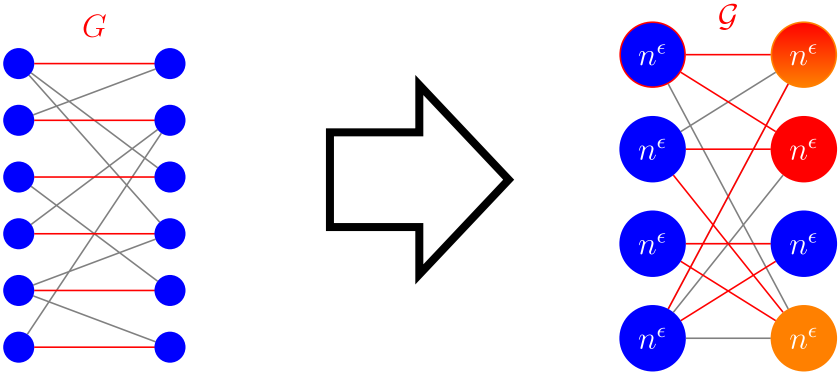

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- $\cdots$
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Conclusion: There exists an n^{ϵ}-approximation algorithm for matchings in $\tilde{O}(n^{2-3\epsilon})$ space.

Lower Bound

Lower Bound for n^ϵ-Approximation

Theorem (Lower bound)

For any $\epsilon \geq 0$, space of $\tilde{\Omega}(n^{2-3\epsilon})$ is necessary for computing an n^ϵ-approximate maximum matching.

- We prove the lower bound for linear sketches.
- Combined with the work of [AHLW16], this provides a tight lower bound for all dynamic graph stream algorithms.

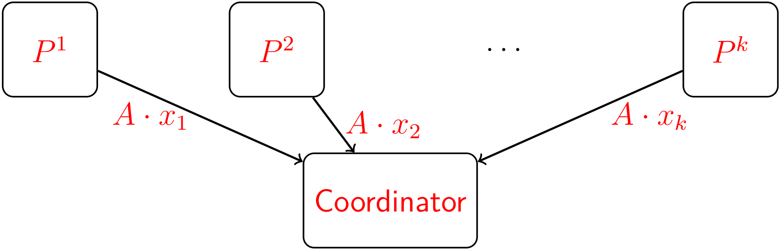
Simultaneous Communication Model

We prove the lower bound using simultaneous communication complexity:

- The input graph is edge partitioned between k players $P^1, \ldots, P^k$.
- There exists another party called the coordinator, with no input.
- Players simultaneously send a message to the coordinator and the coordinator outputs the final matching.
- Communication measure: maximum # of bits send by any player.
- Players have access to public randomness.

Connection to Linear Sketches

If there exists a randomized linear sketch A of size s for a problem P, then the randomized simultaneous communication complexity of P is at most $O(s)$.



$$A \cdot x = A \cdot (x_1 + \ldots + x_k)$$

Hence, a communication lower bound in this model implies an identical space lower bound for linear sketching algorithms.

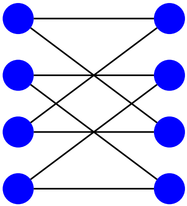
Ruzsa-Szemerédi Graphs

We prove our lower bound using a construction based on Ruzsa-Szemerédi graphs.

Definition ((r, t)-RS graphs)

A graph $G(V, E)$ whose edges can be partitioned into t induced matchings of size r each.

1. Example. A $(2, 4)$-RS graph on 8 vertices:



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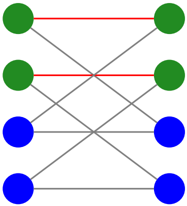

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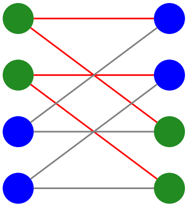

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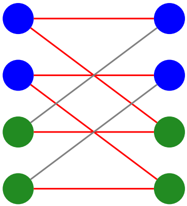

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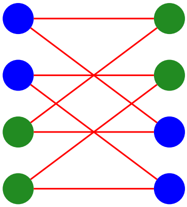

Ruzsa-Szemerédi Graphs

How dense a graph with many large induced matching can be?

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How dense a graph with many large induced matching can be?

Theorem ([AMS12])

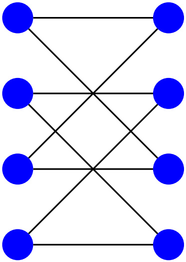
There exists an (r, t)-RS graph on N vertices and $\Omega(N^2)$ edges with $t = N^{1+o(1)}$ induced matchings of size $r = N^{1-o(1)}$.

$\tilde{\Omega}(n^{2-3\epsilon})$ Lower Bound: Distribution

- Parameters:

$$k \approx n^{\epsilon}, r = n^{1-\epsilon-o(1)}, t = n^{1-\epsilon}$$

- Each of the k players is given an (r, t)-RS graph on $n^{1-\epsilon}$ vertices.



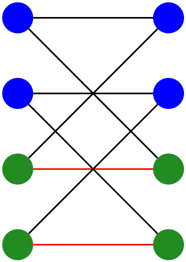
Local view of P^i

$\tilde{\Omega}(n^{2-3\epsilon})$ Lower Bound: Distribution

- Parameters:

$$k \approx n^{\epsilon}, r = n^{1-\epsilon-o(1)}, t = n^{1-\epsilon}$$

- Each of the k players is given an (r, t)-RS graph on $n^{1-\epsilon}$ vertices.
- One induced matching (red edges) of each player's graph is special.



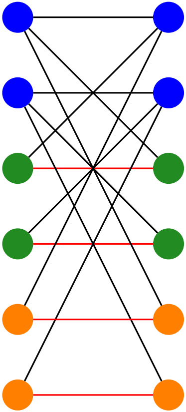
Special matching of P^i

$\tilde{\Omega}(n^{2-3\epsilon})$ Lower Bound: Distribution

- Parameters:

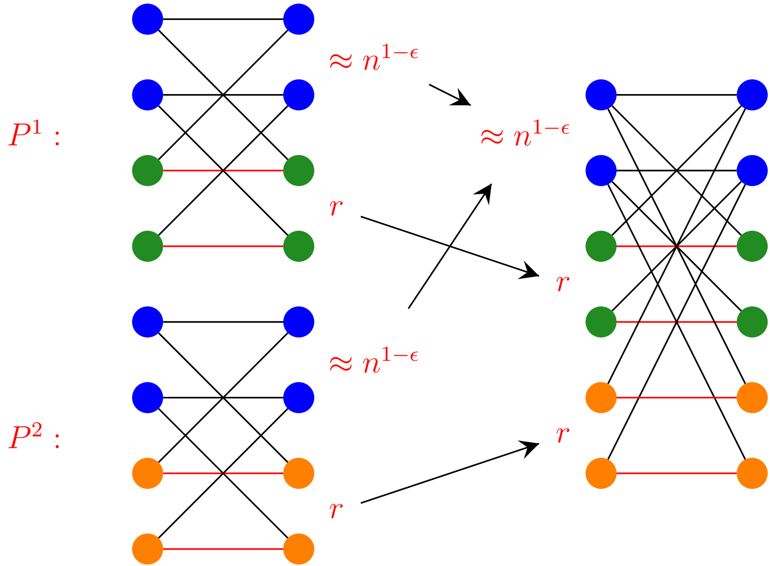
$$k \approx n^{\epsilon}, r = n^{1-\epsilon-o(1)}, t = n^{1-\epsilon}$$

- Each of the k players is given an (r,t)-RS graph on $n^{1-\epsilon}$ vertices.
- One induced matching (red edges) of each player's graph is special.
- Across the players, vertices in the special matchings are unique, while other vertices are shared.



Global view

$\tilde{\Omega}(n^{2-3\epsilon})$ Lower Bound: Distribution

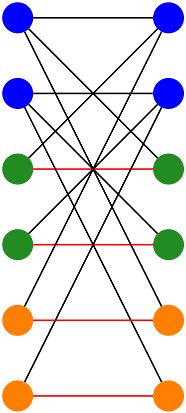


$\tilde{\Omega}(n^{2-3\epsilon})$ Lower Bound: Distribution

Parameters:

$$k \approx n^{\epsilon}, r = n^{1-\epsilon-o(1)}, t = n^{1-\epsilon}$$

- Special matchings are necessary for any large matching.



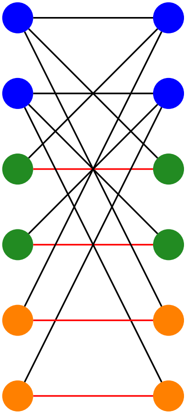
Global view

$\tilde{\Omega}(n^{2-3\epsilon})$ Lower Bound: Distribution

Parameters:

$$k \approx n^{\epsilon}, r = n^{1-\epsilon-o(1)}, t = n^{1-\epsilon}$$

- Special matchings are necessary for any large matching.
- To obtain $o(n^{2-3\epsilon})$ communication, the players have to compress their graph by an $\Omega(n^{\epsilon})$ factor.



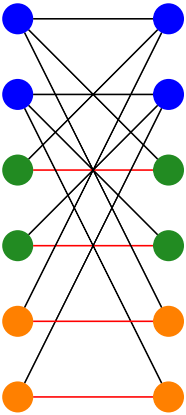
Global view

$\tilde{\Omega}(n^{2-3\epsilon})$ Lower Bound: Distribution

Parameters:

$$k \approx n^{\epsilon}, r = n^{1-\epsilon-o(1)}, t = n^{1-\epsilon}$$

- Special matchings are necessary for any large matching.
- To obtain $o(n^{2-3\epsilon})$ communication, the players have to compress their graph by an $\Omega(n^{\epsilon})$ factor.
- Players are oblivious to the identity of their special matching.



Global view

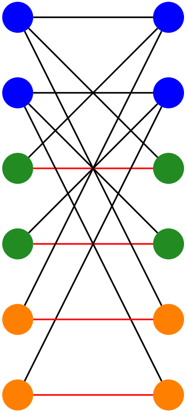
$\tilde{\Omega}(n^{2-3\epsilon})$ Lower Bound: Distribution

Parameters:

$$k \approx n^{\epsilon}, r = n^{1-\epsilon-o(1)}, t = n^{1-\epsilon}$$

- Special matchings are necessary for any large matching.
- To obtain $o(n^{2-3\epsilon})$ communication, the players have to compress their graph by an $\Omega(n^{\epsilon})$ factor.
- Players are oblivious to the identity of their special matching.

Conclusion: Assuming each player sends only $o(n^{2-3\epsilon})$ bits, the coordinator cannot output an n^{ϵ}-approximate maximum matching.



Global view

Conclusion and Open Problems

Space of $\tilde{O}(n^{2-3\epsilon})$ is both sufficient and necessary for computing an n^{ϵ}-approximate maximum matching in dynamic graph streams.

Open question: Can we improve the trivial 2-approximation algorithm for matchings in insertion-only streams?

Questions?

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