Improved Approximation for the Directed Spanner Problem

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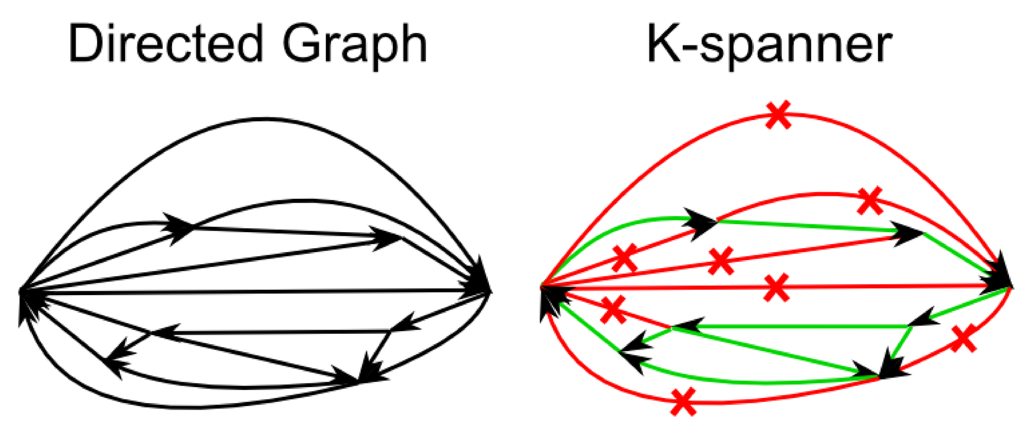
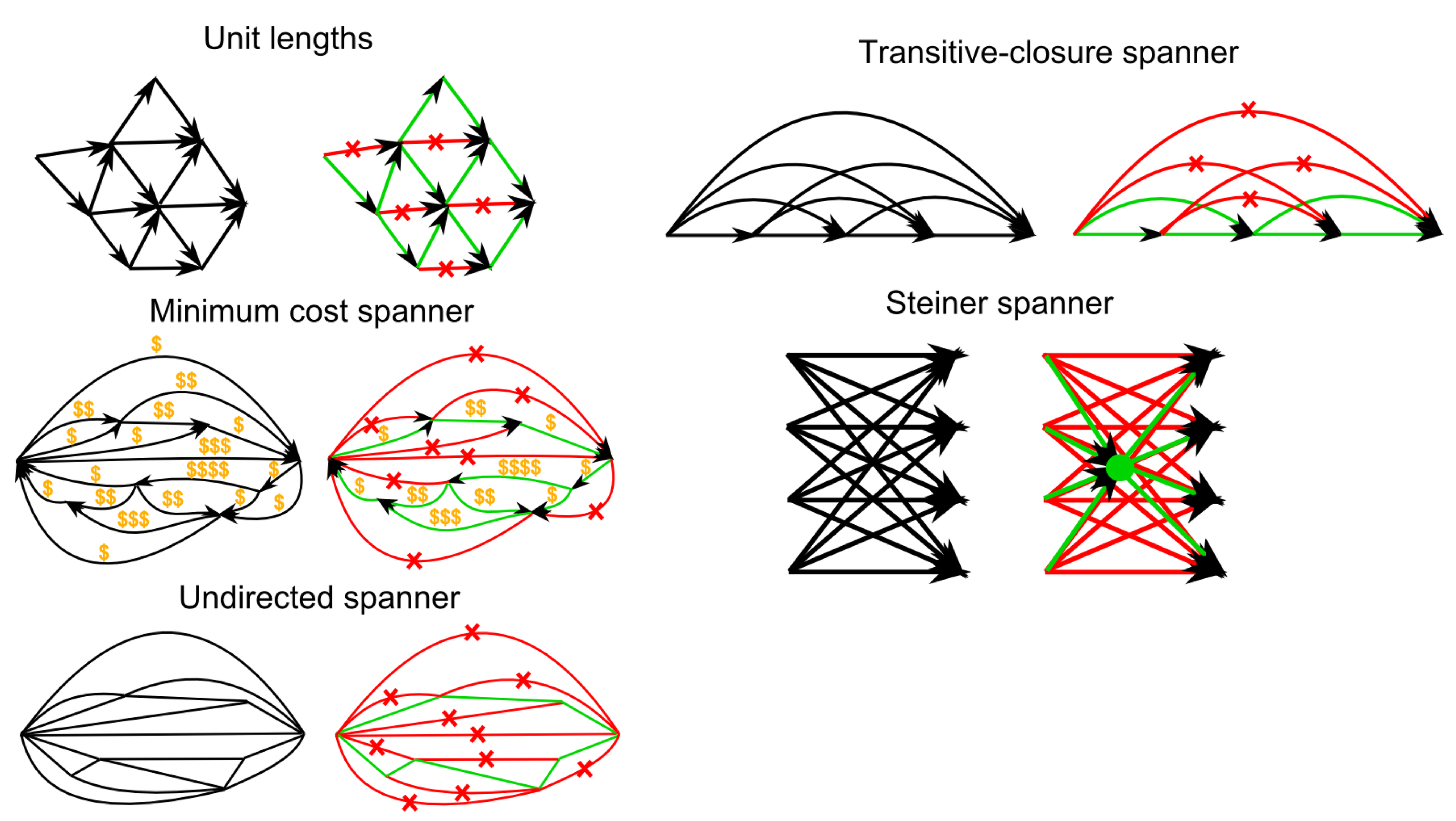
ICALP’11, joint work with P. Berman, A. Bhattacharyya, K. Makarychev, S.Raskhodnikova.

Directed Spanners and Their Friends

K-Spanner -- subset of edges, preserving distances up to a factor k.
Challenge: Find a sparse spanner.



Our result: $\tilde{O}(\sqrt{n})$-approximation randomized algorithm.



Applications of Spanners

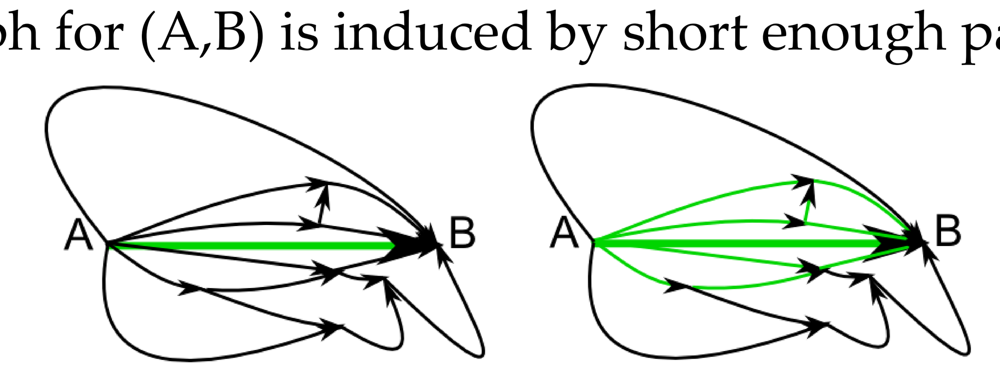
- Efficient routing
- Simulating synchronized protocols in unsynchronized networks
- Parallel , distributed and streaming algorithms for approximating shortest paths
- Algorithms for distance oracles
- Property testing and property reconstruction

Previous Work

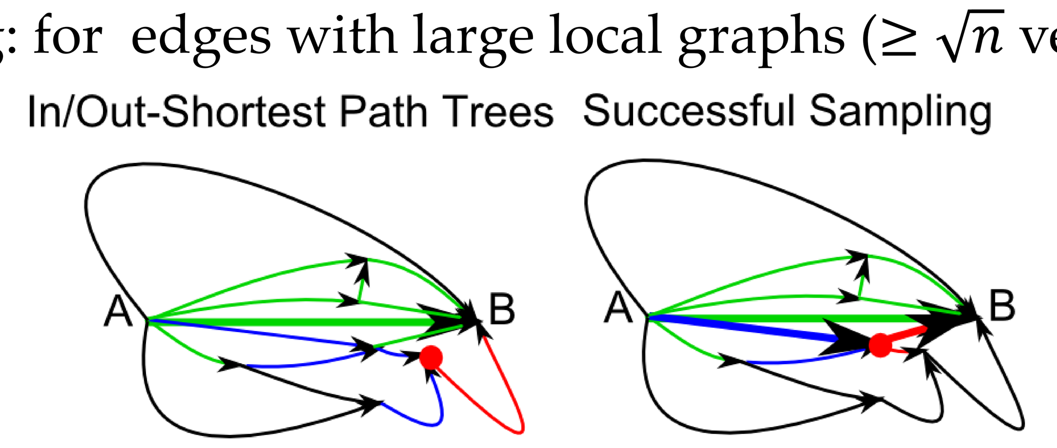
- Related work (approximation): [DK99, BGJRW09, BRR10, DK11]
- $\tilde{O}(n^{2/3})$ Approximation by Dinitz and Krauthgamer, STOC 11 [DK11].
- $2^{\log^{1-\epsilon} n}$ Quasi-NP-hardness, [EP00].
- $\Omega(n^{1/3-\epsilon})$ Integrality gap, [DK11].

Local Graph and Sampling [BGJRW 09, FKN 09]

Local graph for (A,B) is induced by short enough paths.



Sampling: for edges with large local graphs ($\geq \sqrt{n}$ vertices).

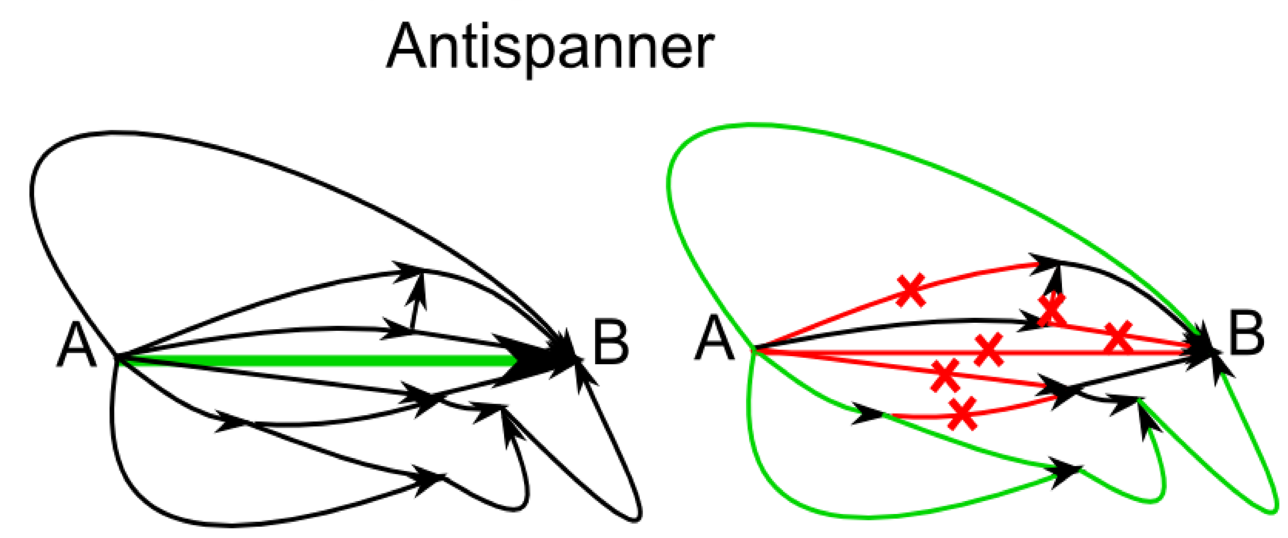


Repeated $\tilde{O}(\sqrt{n})$ times, produces $2(n-1)\tilde{O}(\sqrt{n})$ edges.

Handles all **thick** edges (with $\geq \sqrt{n}$ edges in their local graph) w.h.p.

Antispanners and LP

Antispanner destroys all paths from A to B with stretch at most k.



Linear programming relaxation:

$$\sum_{e \in E} x_e \to min$$

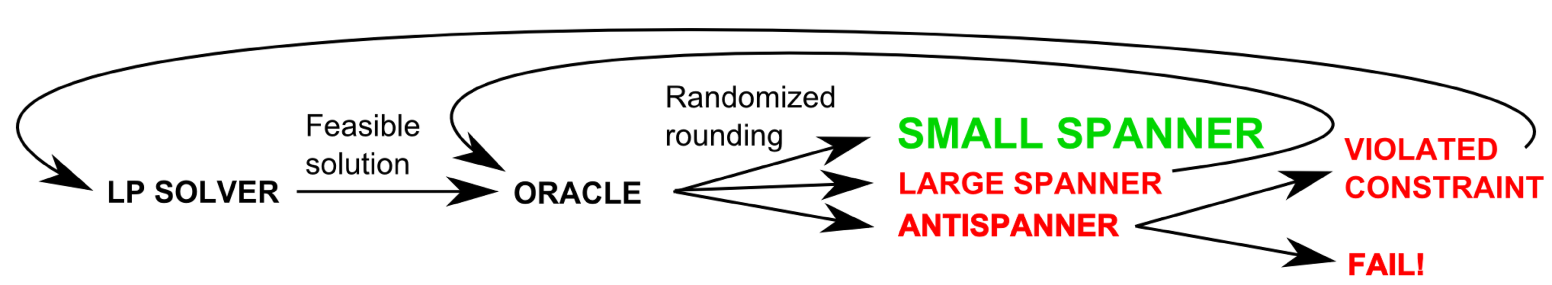
Subject to:

$$\sum_{e \in A} x_e \geq 1$$

for all minimal antispanners A for all thin edges.

Oracle

Oracle: Randomized rounding with a boosting factor $\tilde{O}(\sqrt{n})$.



VIOLATED CONSTRAINT: Number bounded by poly(n).

LARGE SPANNER: Probability is $e^{-\Omega(n\sqrt{n})}$ by Chernoff bound.

FAIL!: One minimal antispanner fails w. p. $\leq e^{-\sqrt{n}\ln n}$.
There are $\leq \sqrt{n}^{\sqrt{n}} = e^{\frac{1}{2}\sqrt{n}\ln n}$ different minimial antispanners.

SMALL SPANNER: Success! (Assuming we’ve guessed OPT.)

Other Improvements

Randomized rounding + similar analysis = improved approximation:

- $\tilde{O}(n^{2/3})$ for Directed Steiner Forest (previous $\tilde{O}(n^{3/4})$, $[FKN09]$).
- $\tilde{O}(n^{2/3})$ for unit-length Minimum Cost Spanner for constant k. (previous $\tilde{O}(n)$ for a more general problem, [DK99])
- $\tilde{O}(n^{1/3})$ for unit-length Undirected 3-Spanner (previous $\tilde{O}(\sqrt{n})$, $[ADDJS93]$).
- $\tilde{O}(n^{1/3})$ for unit-length Directed 3-Spanner (previous $\tilde{O}(\sqrt{n})$, $[BRR11]$).

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