A Model of Computation for MapReduce

Karloff, Suri and Vassilvitskii

$o(n)$ **Big Data Reading Group**

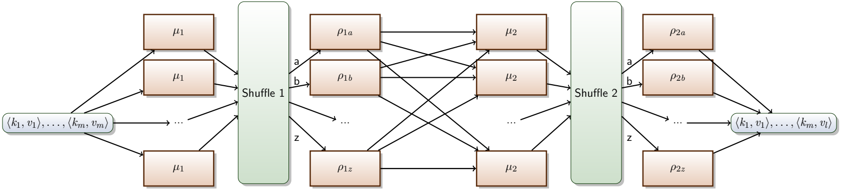
presented by:
Babak Bagheri Hariri

Presentation is prepared based on a presentation by Xing Xie.

Sep 19, 2014

Map Reduce

- A new framework for parallel computing originally developed at Google (before 2004)
- Parallelization of data intensive computation
  - interleaves sequential and parallel computation
  - Tera- and petabytes data set (search engines, internet traffic, bioinformatics, etc)



What is MapReduce (cont.)

Three-stage operations:

- Map-stage: mapper operates on a single pair $\langle \text{key}, \text{value} \rangle$, outputs any number of new pairs $\langle \text{key}', \text{value}' \rangle$;
  - operation is stateless (parallel)
- Shuffle-stage: all values that are associated to an individual key are sent to a single machine (done by the system)
- Reduce-stage: reducer operates on the all the values and outputs a multiset of $\langle \text{key}, \text{value} \rangle$.
  - stage can only start when all Map operations are done.

An example: k^{th} frequency moment of a large data set

- Input: a finite string of symbols $s = a_1, a_2, \ldots, a_n$;
- Let $f(x)$ be the frequency of the symbol x,
  - note: $\sum_{x \in s} f(x) = n$;
- Want to compute $\sum_{x \in s} f^k(x)$;

example:

$s = 1, 1, 2, 4, 1$
$f^1(x) = 3^1 + 1^1 + 1^1 = 5;$
$f^2(x) = 3^2 + 1^2 + 1^2 = 11.$

An example (cont.)

- Input to each mapper: $\langle i, x_i \rangle$
  - $\mu_1(\langle i, x_i \rangle) = \langle x_i, i \rangle$ (i is the index).
- Input to each reducer: $\langle x_i, \{i_1, i_2, \dots, i_m\} \rangle$
  - $\rho_1(\langle x_i, \{i_1, i_2, \dots, i_m\} \rangle) = \langle x_i, m^k \rangle$;
- Map the values to a single reserved symbol '\$'
  - $\mu_2(\langle x_i, v \rangle) = \langle \$, v \rangle$;
- A single reducer for summing up the values:
  - $\rho_2(\langle \$, \{v_1, \dots, v_l\} \rangle) = \langle \$, \sum v_i \rangle$.

Formal Definition

- The input is a finite sequence of pairs of binary strings $\langle \text{key}, \text{value} \rangle$;
  - $U_0 = \langle k_1, v_1 \rangle, \cdots \langle k_m, v_m \rangle$
- A MapReduce program consists of a finite sequence of mappers and reducers;
  - $\mu_1, \rho_1, \mu_2, \rho_2, \ldots, \mu_l, \rho_l$;
- Execution: For $r = 1, 2, \ldots, l$
  - (Map) feed each $\langle k, v \rangle$ in U_{r-1} to mapper μ_r.
    - ⋆ Let the output be U'_r;
  - for each k
    - ⋆ (Shuffle) $V_{k,r}$ is the multiset of values v, s.t., $\langle k, v_i \rangle \in U_{r-1}$;
    - ⋆ feed k and $V_{k,r}$ to a separate instance of ρ_r;
    - ⋆ (Reduce) Let U_r be the multiset of $\langle \text{key}, \text{value} \rangle$ generated by all instances of ρ_r.
  - Output U_l.

The MapReduce Class ($\mathcal{MRC}$)

- On input I with size: $n = \sum_{\langle k,v \rangle \in I} (|k| + |v|)$
  - Memory: Memory: each mapper/reducer uses $O(n^{1-\epsilon})$ space;
  - Machines: There are $O(n^{1-\epsilon})$ machines available;
  - Time: each machine runs in time polynomial in n, (not in the length of the input they receive);
  - Randomized algorithms for map and reduce;
  - The algorithm outputs the correct answer with probability at least $3/4$;
  - Rounds: Shuffle is expensive:
    - ★ $\mathcal{MRC}^i$. number of rounds $= O(\log^i n)$
- $\mathcal{DMRC}$: the deterministic variant.

Lemma

For all rounds of an algorithm in $\mathcal{MRC}$, it is possible to partition the output of the mappers among reducers such that the memory restrictions of $\mathcal{MRC}$ would not be violated.

Recall the Frequency Moments Algorithem

Does this algorithm fit in the restrictions of $\mathcal{MRC}$?

- $\mu_1(\langle i, x_i \rangle) = \langle x_i, i \rangle$;
- $\rho_1(\langle x_i, \{i_1, i_2, \dots, i_m\} \rangle) = \langle x_i, m^k \rangle$;
- $\mu_2(\langle x_i, v \rangle) = \langle \$, v \rangle$;
- $\rho_2(\langle \$, \{v_1, \dots, v_l\} \rangle) = \langle \$, \sum v_i \rangle$.

Consider the input $I = \langle 1, a \rangle, \langle 2, a \rangle, \dots, \langle n, a \rangle$.

Comparing $\mathcal{MRC}$ with other Complexity Classes

Easy relation: $\mathcal{MRC} \subseteq \mathcal{P}$;

Lemma

If $\mathcal{NC} \neq \mathcal{P}$, then $\mathcal{DMRC} \not\subseteq \mathcal{NC}$;

Proof idea: There exists a $\mathcal{P}$-complete problem solvable in $\mathcal{DMRC}$:

- Padded Circuite Value Problem (PCV) is a $\mathcal{P}$-complete problem;
- For a given PCV problem with input size n, append the input with $n^2 - n$ special character $\sharp$;
- The problem is in $\mathcal{DMRC}$;
- But it cannot be in $\mathcal{NC}$; otherwise, we would have $\mathcal{NC} = \mathcal{P}$!

Open question: $\mathcal{P} \subseteq \mathcal{DMRC}$?

Example: Finding an MST

Problem:

Find the Minimum Spanning Tree (MST) of a dense graph.

The algorithm:

- Randomly partition the vertices of G into k parts;
- For each pair of vertex sets, find the MST of the subgraph induce by these two sets;
- Take the union H of all the edges in the MST of each pair;
- Compute an MST of H

Theorem

the MST tree of H is an MST of G

Proof idea: we did not discard any relevant edge when sparsifying the input graph G

Finding an MST (cont.)

Why the algorithm is in MRC?

- Let $N = |V|$ and $m = |E| = N^{1+c}$, for $0 < c \leq 1$;
- So input size n satisfies $n = N^{1+c}$;
- Pick $k = N^{c/2}$;

Lemma

With high probability, each subgraph has size $N^{1+c/2}$.

- so the input to any reducer is $n^{1-\epsilon}$;
- the size of H is also in $n^{1-\epsilon}$.

Functions Lemma

A very useful building block for designing MapReduce algorithms:

$\mathcal{MRC}$-parallelizable function

Let S be a finite set. We say a function f on S is $\mathcal{MRC}$-parallelizable if there are functions g and h so that the followings hold:

- For all partition of S, $S = T_1 \cup T_2 \cup \cdots \cup T_k$, f can be written as: $f(S) = h(g(T_1), g(T_2), \ldots, g(T_k))$;
- g and h each can be represented in $O(\log n)$ bits;
- g and h can be computed in time polynomial in $|S|$;
- all possible outputs of g can be expressed in $O(\log n)$ bits.

Application of Functions Lemma (1): the Frequency Moments Algorithem

Input $\mathcal{I} = \{\langle 1, l_1 \rangle, \ldots, \langle m, l_m \rangle\}$;

- define $f_{k,l}(\mathcal{I}) = |\text{occurrences of the element } l \text{ in the input } \mathcal{I}|^k$;
- k^{th}-frequency moment of $\mathcal{I}$ is $\sum_l f_{k,l}(l)$;
- $f_{k,l}(l)$ is $\mathcal{MRC}$-parallelizable:
  - $g(t_1, \ldots, t_n) = n$;
  - $h(i_1, \ldots, i_r) = (i_1 + \ldots, +i_r)^k$.

Application of the Functions Lemma (2): $s - t$ connectivity

$s - t$ Connectivity Problem:

Given a graph G and two nodes, are they connected in G?

- for dense graphs: easy, powering adjacency matrix;
- Sparse graphs?

A log$n$-round MapReduce algorithm for $s - t$ connectivity

- Initially every node is active;
- For $i = 1, 2, \ldots, O(\log n)$ do
  - Each active node becomes a leader with probability $1/2$;
  - For each non-leader active node u, find a node v in the neighbor of u's current connected component
  - If the connected component of v is non-empty, then u become passive and re-label each node in u's connected component with v's label.
- Output true if s and t have the same label, false otherwise.

Thanks!