

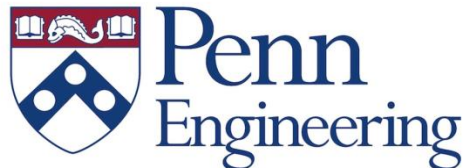
Open Problems in L_p -Testing

See our joint work with P. Berman and S.
Raskhodnikova (STOC'14).

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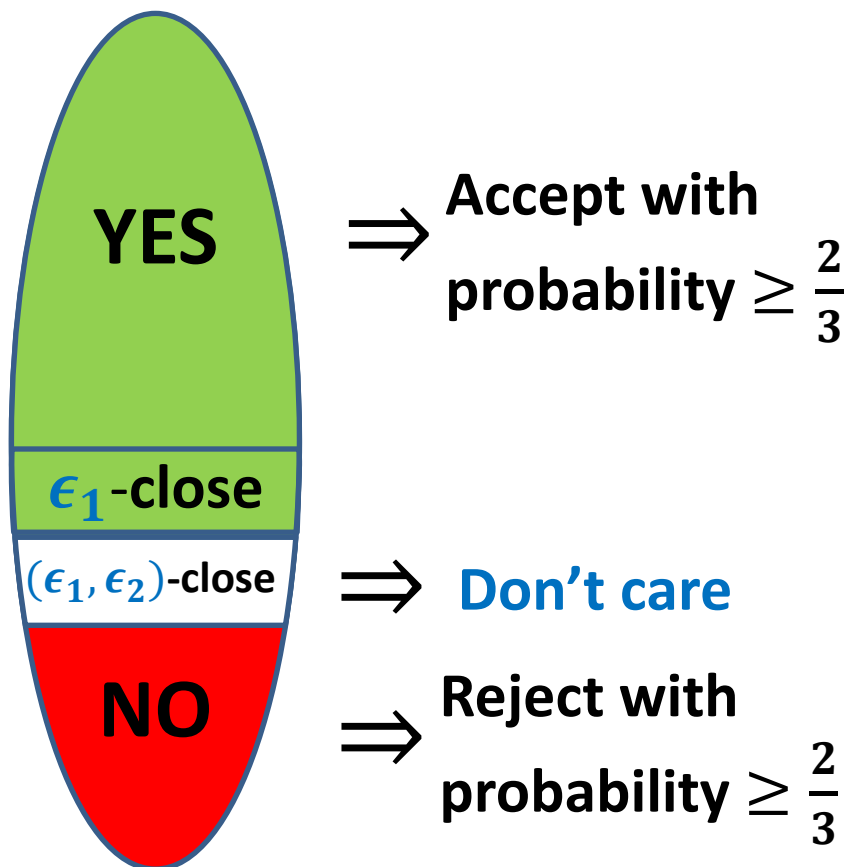
<http://grigory.us>



Tolerant “ L_1 Property Testing”

- $f: \{1, \dots, n\}^d \rightarrow [0, 1]$
- \mathcal{P} = class of functions (monotone, convex, Lipschitz)
- $dist_1(f, \mathcal{P}) = \frac{\min_{g \in \mathcal{P}} |f - g|_1}{n^d}$
- ϵ -close: $dist_1(f, \mathcal{P}) \leq \epsilon$
- $\epsilon_1 = 0 \Rightarrow$ Non-tolerant
- $\epsilon_1 \neq 0 \Rightarrow$ Tolerant

Tolerant “ L_1 Property Tester”



Known non-tolerant L_1 -Testers

- Monotonicity: $f: [n]^d \rightarrow [0,1]$:

$$O\left(\frac{d}{\epsilon} \log \frac{d}{\epsilon}\right) \text{ (see BRY'14 for lower bounds)}$$

- Lipschitz property $f: [n]^d \rightarrow [0,1]$:

$$\Theta\left(\frac{d}{\epsilon}\right) \text{ (tight)}$$

- Convexity $f: [n]^d \rightarrow [0,1]$:

$$O\left(\epsilon^{-\frac{d}{2}} + \frac{1}{\epsilon}\right) \text{ (tight for } d \leq 2)$$

- Submodularity $f: \{0,1\}^d \rightarrow [0,1]$

$$2^{\tilde{O}\left(\frac{1}{\epsilon}\right)} + \text{poly}\left(\frac{1}{\epsilon}\right) \log d \text{ [Feldman, Vondrak 13, ...]}$$

Open Problem #1

- Complexity for non-tolerant L_1 -testing convexity grows exponentially with d

Is there an L_1 -testing algorithm for convexity with subexponential dependence on the dimension?

- Why is it hard?
 - Relevant reference: [Rademacher, Vempala'04]
 - Restrictions on 1-dimensional axis-parallel lines don't help (need exponentially many)
 - Can 2-dimensional restrictions help?

L_1 -Testing for Convex Optimization

- **Theory:** Convergence rates of gradient descent methods depends on:
 - Convexity / strong convexity constant
 - Lipschitz constant of the derivative
- **Practice:**
 - Q: How to pick learning rate in ML packages?
 - A: Set 0.01 and hope it converges fast
- Even non-tolerant L_1 -testers can be used to sanity check convexity/Lipschitzness



Known tolerant L_1 -testers

- Monotonicity in 1D

$$O\left(\frac{\varepsilon_2}{(\varepsilon_2 - \varepsilon_1)^2}\right)$$

- Monotonicity in 2D

$$\tilde{O}\left(\frac{1}{(\varepsilon_2 - \varepsilon_1)^4}\right)$$

Open Problem #2

- Only have tolerant monotonicity for $d = 1, 2$.
Tolerant testers for higher dimensions?