Approximation Algorithms Workshop	June 13-17, 2011, Princeton

Open Problems Session

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1 Directed k-Spanner Problem

Proposed by Grigory Yaroslavtsev, for more details see slides here and a paper here

Problem statement Let $G(V, E)$ be a weighted *directed* graph. A k-spanner is a subset of edges of G, which preserves distances in the original graph up to a factor k. Formally, a k-spanner is a graph $G_H(V, E_H)$, where $E_H \subseteq E$ and $\forall\ (u, v) \in E$ we have $d_{G_H}(u, v) \leq k \cdot d_G(u, v)$.

We want to find a directed $k-$spanner which *minimizes* the number of edges $|E_H|$. What is the best approximation factor that we can get?

Most recent previous work

- ...
- Dinitz and Krauthgamer (STOC 2011) gave a $\tilde{O}(n^{\frac{2}{3}})$ approximation and showed an integrality gap of $\Omega(n^{\frac{1}{3}-\epsilon})$.
- This was improved to $\tilde{O}(\sqrt{n})$ by Berman, Bhattacharya, Makarychev, Raskhodnikova and Yaroslavtsev (ICALP 2011).
- Hardness: Elkin and Peleg (STOC 2000) show that it is quasi-NP-hard to approximate with ratio better than $2^{log^{1-\epsilon} n}$.
- Integrality gap: $\Omega(n^{1/3-\epsilon})$ (for constant k) by Dinitz and Krauthgamer.

Questions :

- Can we beat this ratio?
- Current method is randomized rounding of an LP relaxation, combined with sampling. What other techniques can we use?
- What is a natural online setting for this problem?

Some comments by the listeners

- **Q:** How about undirected spanners?

  A: They are very different, because girth arguments work there.

- **Q:** How about directed spanners of minimum cost?

  A: The best result is by Dodis and Khanna (STOC 1999), who give $\tilde{O}(n)$-approximation.

- In above problem we had fixed k and wanted to minimize $|E_M|$. We can also consider the problem where we have a bound on $|E_M|$ and then want to minimize k.

- **Q:**Is there an example, when there is a sparse directed spanner of a dense graph?

  A: For every k it is easy to construct a graph with $\Omega(n^2)$

- How well can we approximate the size of the sparsest $2k$-spanner, relative to the size of the sparsest k-spanner?