1 Directed k-Spanner Problem

Proposed by Grigory Yaroslavtsev, for more details see slides and a paper.

Problem statement Let $G(V, E)$ be a weighted directed graph. A $k$-spanner is a subset of edges of $G$, which preserves distances in the original graph up to a factor $k$. Formally, a $k$-spanner is a graph $G_H(V, E_H)$, where $E_H \subseteq E$ and $\forall (u, v) \in E$ we have $d_{G_H}(u, v) \leq k \cdot d_G(u, v)$.

We want to find a directed $k$-spanner which minimizes the number of edges $|E_H|$. What is the best approximation factor that we can get?

Most recent previous work

- Dinitz and Krauthgamer (STOC 2011) gave a $\tilde{O}(n^{3/4})$ approximation and showed an integrality gap of $\Omega(n^{1/3-\epsilon})$.
- This was improved to $\tilde{O}(\sqrt{n})$ by Berman, Bhattacharya, Makarychev, Raskhodnikova and Yaroslavtsev (ICALP 2011).
- Hardness: Elkin and Peleg (STOC 2000) show that it is quasi-NP-hard to approximate with ratio better than $2^{\log^{1/3} n}$.
- Integrality gap: $\Omega(n^{1/3-\epsilon})$ (for constant $k$) by Dinitz and Krauthgamer.

Questions:

- Can we beat this ratio?
- Current method is randomized rounding of an LP relaxation, combined with sampling. What other techniques can we use?
- What is a natural online setting for this problem?

Some comments by the listeners

- Q: How about undirected spanners?
  A: They are very different, because girth arguments work there.
• **Q:** How about directed spanners of minimum cost?
  **A:** The best result is by Dodis and Khanna (STOC 1999), who give $\tilde{O}(n)$-approximation.

• In above problem we had fixed $k$ and wanted to minimize $|E_M|$. We can also consider the problem where we have a bound on $|E_M|$ and then want to minimize $k$.

• **Q:** Is there an example, when there is a sparse directed spanner of a dense graph?
  **A:** For every $k$ it is easy to construct a graph with $\Omega(n^2)$

• How well can we approximate the size of the sparsest $2k$-spanner, relative to the size of the sparsest $k$-spanner?